7-1 Ratios and Proportions

Objective To write ratios and solve proportions

The year the team had the most wins is not necessarily the year in which it had the best record.

Getting Ready!

The table at the right gives the wins and losses of a baseball team. In which year(s) did the team have the best record? Explain.

Year	Wins	Losses
1890	60	24
1930	110	44
1970	110	52
2010	108	54

In the Solve It, you compared two quantities for four years.

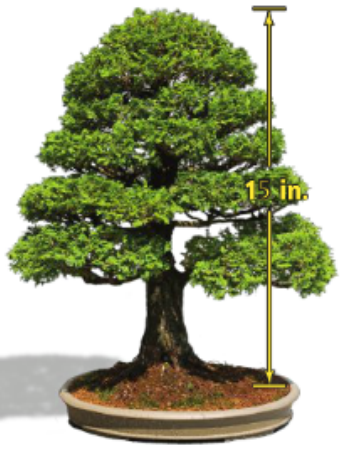
Essential Understanding You can write a *ratio* to compare two quantities.

Lesson Vocabulary
- ratio
- extended ratio
- proportion
- extremes
- means
- Cross Products Property

A **ratio** is a comparison of two quantities by division. You can write the ratio of two numbers a and b, where $b \neq 0$, in three ways: $\frac{a}{b}$, $a : b$, and a to b. You usually express a and b in the same unit and write the ratio in simplest form.

Problem 1 Writing a Ratio

Bonsai Trees **The bonsai bald cypress tree is a small version of a full-size tree. A Florida bald cypress tree called the Senator stands 118 ft tall. What is the ratio of the height of the bonsai to the height of the Senator?**



Think

How can you write the heights using the same unit?
You can convert the height of the Senator to inches or the height of the bonsai tree to feet.

Express both heights in the same unit. To convert 118 ft to inches, multiply by the conversion factor $\frac{12 \text{ in.}}{1 \text{ ft}}$.

$$118 \text{ ft} = \frac{118 \text{ ft}}{1} \cdot \frac{12 \text{ in.}}{1 \text{ ft}} = (118 \cdot 12) \text{ in.} = 1416 \text{ in.}$$

Write the ratio as a fraction in simplest form.

$$\begin{matrix}\text{height of bonsai} \rightarrow \\ \text{height of Senator} \rightarrow\end{matrix} \frac{15 \text{ in.}}{118 \text{ ft}} = \frac{15 \text{ in.}}{1416 \text{ in.}} = \frac{(3 \cdot 5) \text{ in.}}{(3 \cdot 472) \text{ in.}} = \frac{5}{472}$$

The ratio of the height of the bonsai to the height of the Senator is $\frac{5}{472}$ or $5 : 472$.

Got It? **1.** A bonsai tree is 18 in. wide and stands 2 ft tall. What is the ratio of the width of the bonsai to its height?

Problem 2 Dividing a Quantity Into a Given Ratio

Fundraising Members of the school band are buying pots of tulips and pots of daffodils to sell at their fundraiser. They plan to buy 120 pots of flowers. The ratio $\frac{\text{number of tulip pots}}{\text{number of daffodil pots}}$ will be $\frac{2}{3}$. How many pots of each type of flower should they buy?

Plan

How do you write expressions for the numbers of pots?
Multiply the numerator 2 and the denominator 3 by the factor x. $\frac{2x}{3x} = \frac{2}{3}$.

Think

If the ratio $\frac{\text{number of tulip pots}}{\text{number of daffodil pots}}$ is $\frac{2}{3}$, it must be in the form $\frac{2x}{3x}$.

The total number of flower pots is 120. Use this fact to write an equation. Then solve for x.

Substitute 24 for x in the expressions for the numbers of pots.

Write the answer in words.

Write

Let $2x$ = the number of tulip pots.

Let $3x$ = the number of daffodil pots.

$$2x + 3x = 120$$
$$5x = 120$$
$$x = 24$$

$$2x = 2(24) = 48$$
$$3x = 3(24) = 72$$

The band members should buy 48 tulip pots and 72 daffodil pots.

Got It? **2.** The measures of two supplementary angles are in the ratio 1 : 4. What are the measures of the angles?

An **extended ratio** compares three (or more) numbers. In the extended ratio $a : b : c$, the ratio of the first two numbers is $a : b$, the ratio of the last two numbers is $b : c$, and the ratio of the first and last numbers is $a : c$.

Problem 3 Using an Extended Ratio

The lengths of the sides of a triangle are in the extended ratio 3 : 5 : 6. The perimeter of the triangle is 98 in. What is the length of the longest side?

Think

How do you use the solution of the equation to answer the question?
Substitute the value for x in the expression for the length of the longest side.

Sketch the triangle. Use the extended ratio to label the sides with expressions for their lengths.

$3x + 5x + 6x = 98$	The perimeter is 98 in.
$14x = 98$	Simplify.
$x = 7$	Divide each side by 14.

The expression that represents the length of the longest side is $6x$. $6(7) = 42$, so the length of the longest side is 42 in.

Got It? **3.** The lengths of the sides of a triangle are in the extended ratio 4 : 7 : 9. The perimeter is 60 cm. What are the lengths of the sides?

Essential Understanding If two ratios are equivalent, you can write an equation stating that the ratios are equal. If the equation contains a variable, you can solve the equation to find the value of the variable.

An equation that states that two ratios are equal is called a **proportion.** The first and last numbers in a proportion are the **extremes.** The middle two numbers are the **means.**

$$\underset{\text{extremes}}{2} : 3 = 4 : \underset{\text{extremes}}{6}$$
$$\underset{\text{means}}{\uparrow} \quad \underset{\text{means}}{\uparrow}$$

extremes → 2 and 6; means → 3 and 4

take note

Key Concept Cross Products Property

Words	Symbols	Example
In a proportion, the product of the extremes equals the product of the means.	If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = bc$.	$\frac{2}{3} = \frac{4}{6}$ $2 \cdot 6 = 3 \cdot 4$ $12 = 12$

Here's Why It Works Begin with $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$.

$bd \cdot \frac{a}{b} = \frac{c}{d} \cdot bd$	Multiply each side of the proportion by bd.
$\frac{\cancel{b}d}{1} \cdot \frac{a}{\cancel{b}} = \frac{c}{\cancel{d}} \cdot \frac{b\cancel{d}}{1}$	Divide the common factors.
$ad = bc$	Simplify.

ONLINE PROBLEM

Problem 4 Solving a Proportion

Algebra **What is the solution of each proportion?**

A $\frac{6}{x} = \frac{5}{4}$

$6(4) = 5x$	Cross Products Property
$24 = 5x$	Simplify.
$x = \frac{24}{5}$	Solve for the variable.

The solution is $\frac{24}{8}$ or 4.8.

Think

Does the solution check?

$\frac{6}{\frac{24}{5}} \stackrel{?}{=} \frac{5}{4}$

$6 \cdot 4 \stackrel{?}{=} \frac{24}{5} \cdot 5$

$24 = 24$ ✓

B $\frac{y+4}{9} = \frac{y}{3}$

$3(y + 4) = 9y$	
$3y + 12 = 9y$	
$12 = 6y$	
$y = 2$	

The solution is 2.

Got It? **4.** What is the solution of each proportion?

a. $\frac{9}{2} = \frac{a}{14}$ **b.** $\frac{15}{m+1} = \frac{3}{m}$

Using the Properties of Equality, you can rewrite proportions in equivalent forms.

take note

Key Concept Properties of Proportions

a, b, c, and d do not equal zero.

Property

(1) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{b}{a} = \frac{d}{c}$.

(2) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a}{c} = \frac{b}{d}$.

(3) $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a + b}{b} = \frac{c + d}{d}$.

How to Apply It

Write the reciprocal of each ratio.

$\frac{2}{3} = \frac{4}{6}$ becomes $\frac{3}{2} = \frac{6}{4}$.

Switch the means.

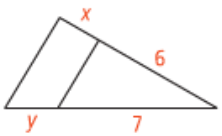
$\frac{2}{3} \times \frac{4}{6}$ becomes $\frac{2}{4} = \frac{3}{6}$.

In each ratio, add the denominator to the numerator.

$\frac{2}{3} = \frac{4}{6}$ becomes $\frac{2 + 3}{3} = \frac{4 + 6}{6}$.

Problem 5 Writing Equivalent Proportions

In the diagram, $\frac{x}{6} = \frac{y}{7}$. What ratio completes the equivalent proportion $\frac{x}{y} = \frac{\blacksquare}{\blacksquare}$? Justify your answer.



Plan

How do you decide which property of proportions applies? Look at how the positions of the known parts of the incomplete proportion relate to their positions in the original proportion.

Method 1

$\frac{x}{6} = \frac{y}{7}$

$\frac{x}{y} = \frac{6}{7}$ Property of Proportions (2)

Method 2

$\frac{x}{6} = \frac{y}{7}$

$7x = 6y$ Cross Products Property

$\frac{7x}{7y} = \frac{6y}{7y}$ To solve for $\frac{x}{y}$, divide each side by $7y$.

$\frac{x}{y} = \frac{6}{7}$ Simplify.

The ratio that completes the proportion is $\frac{6}{7}$.

Got It? **5.** For parts (a) and (b), use the proportion $\frac{x}{6} = \frac{y}{7}$. What ratio completes the equivalent proportion? Justify your answer.

a. $\frac{6}{x} = \frac{\blacksquare}{\blacksquare}$

b. $\frac{\blacksquare}{\blacksquare} = \frac{y + 7}{7}$

c. Reasoning Explain why $\frac{6}{x - 6} = \frac{7}{y - 7}$ is an equivalent proportion to $\frac{x}{6} = \frac{y}{7}$.

Lesson Check

Do you know HOW?

1. To the nearest millimeter, a cell phone is 84 mm long and 46 mm wide. What is the ratio of the width to the length?
2. Two angle measures are in the ratio 5 : 9. Write expressions for the two angle measures in terms of the variable x.
3. What is the solution of the proportion $\frac{20}{z} = \frac{5}{3}$?
4. For $\frac{a}{7} = \frac{13}{b}$ complete each equivalent proportion.
   a. $\frac{a}{\blacksquare} = \frac{7}{\blacksquare}$ b. $\frac{a-7}{7} = \frac{\blacksquare}{\blacksquare}$ c. $\frac{7}{a} = \frac{\blacksquare}{\blacksquare}$

Do you UNDERSTAND?

5. **Vocabulary** What is the difference between a ratio and a proportion?
6. **Open-Ended** The lengths of the sides of a triangle are in the extended ratio 3 : 6 : 7. What are two possible sets of side lengths, in inches, for the triangle?
7. **Error Analysis** What is the error in the solution of the proportion shown at the right?

$$\frac{7}{3} = \frac{4}{x}$$
$$28 = 3x$$
$$\frac{28}{3} = x$$

8. What is a proportion that has means 6 and 18 and extremes 9 and 12?

Practice and Problem-Solving Exercises

A Practice

Write the ratio of the first measurement to the second measurement. See Problem 1.

9. length of a tennis racket: 2 ft 4 in.
   length of a table tennis paddle: 10 in.
10. height of a table tennis net: 6 in.
    height of a tennis net: 3 ft
11. diameter of a table tennis ball: 40 mm
    diameter of a tennis ball: 6.8 cm
12. length of a tennis court: 26 yd
    length of a table tennis table: 9 ft

13. **Baseball** A baseball team played 154 regular season games. The ratio of the number of games they won to the number of games they lost was $\frac{5}{2}$. How many games did they win? How many games did they lose? See Problem 2.
14. The measures of two supplementary angles are in the ratio 5 : 7. What is the measure of the larger angle?
15. The lengths of the sides of a triangle are in the extended ratio 6 : 7 : 9. The perimeter of the triangle is 88 cm. What are the lengths of the sides? See Problem 3.
16. The measures of the angles of a triangle are in the extended ratio 4 : 3 : 2. What is the measure of the largest angle?

Algebra **Solve each proportion.** See Problem 4.

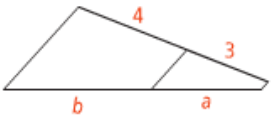
17. $\frac{1}{3} = \frac{x}{12}$
18. $\frac{9}{5} = \frac{3}{x}$
19. $\frac{4}{x} = \frac{5}{9}$
20. $\frac{y}{10} = \frac{15}{25}$
21. $\frac{9}{24} = \frac{12}{n}$
22. $\frac{11}{14} = \frac{b}{21}$
23. $\frac{3}{5} = \frac{6}{x+3}$
24. $\frac{y+7}{9} = \frac{8}{5}$
25. $\frac{5}{x-3} = \frac{10}{x}$
26. $\frac{n+4}{8} = \frac{n}{4}$

In the diagram, $\frac{a}{b} = \frac{3}{4}$. Complete each statement. Justify your answer.

See Problem 5.

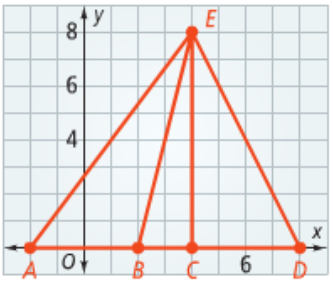
27. $\frac{b}{a} = \frac{\blacksquare}{\blacksquare}$ 28. $4a = \blacksquare$ 29. $\frac{\blacksquare}{\blacksquare} = \frac{b}{4}$

30. $\frac{\blacksquare}{\blacksquare} = \frac{7}{4}$ 31. $\frac{a+b}{b} = \frac{\blacksquare}{\blacksquare}$ 32. $\frac{b}{\blacksquare} = \frac{4}{\blacksquare}$



B Apply

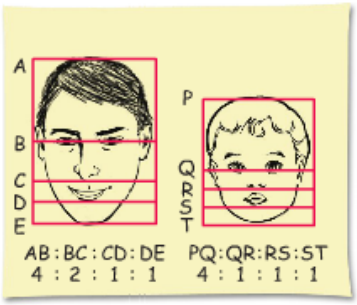
Coordinate Geometry Use the graph. Write each ratio in simplest form.



33. $\frac{AC}{BD}$ 34. $\frac{AE}{EC}$ 35. slope of $\overline{EB}$ 36. slope of $\overline{ED}$

37. **Think About a Plan** The area of a rectangle is 150 in.^{2}. The ratio of the length to the width is 3 : 2. Find the length and the width.
- What is the formula for the area of a rectangle?
- How can you use the given ratio to write expressions for the length and width?

Art To draw a face, you can sketch the head as an oval and then lightly draw horizontal lines to help locate the eyes, nose, and mouth. You can use the extended ratios shown in the diagrams to help you place the lines for an adult's face or for a baby's face.



38. If $AE = 72$ cm in the diagram, find AB, BC, CD, and DE.

39. You draw a baby's head as an oval that is 21 in. from top to bottom.
- **a.** How far from the top should you place the line for the eyes?
- **b.** Suppose you decide to make the head an adult's head. How far up should you move the line for the eyes?

Algebra Solve each proportion.

40. $\frac{1}{7y-5} = \frac{2}{9y}$ 41. $\frac{4a+1}{7} = \frac{2a}{3}$ 42. $\frac{5}{x+2} = \frac{3}{x+1}$ 43. $\frac{2b-1}{4} = \frac{b-2}{12}$

44. The ratio of the length to the width of a rectangle is 9 : 4. The width of the rectangle is 52 mm. Write and solve a proportion to find the length.

45. **Open-Ended** Draw a quadrilateral that satisfies this condition: The measures of the consecutive angles are in the extended ratio 4 : 5 : 4 : 7.

46. **Reasoning** The means of a proportion are 4 and 15. List all possible pairs of positive integers that could be the extremes of the proportion.

47. **Writing** Describe how to use the Cross Products Property to determine whether $\frac{10}{26} = \frac{16}{42}$ is a true proportion.

48. **Reasoning** Explain how to use two different properties of proportions to change the proportion $\frac{3}{4} = \frac{12}{16}$ into the proportion $\frac{12}{3} = \frac{16}{4}$.

Complete each statement. Justify your answer.

49. If $4m = 9n$, then $\frac{m}{n} = \frac{\blacksquare}{\blacksquare}$.

50. If $\frac{30}{t} = \frac{18}{r}$, then $\frac{t}{r} = \frac{\blacksquare}{\blacksquare}$.

51. If $\frac{a+5}{5} = \frac{b+2}{2}$, then $\frac{a}{5} = \frac{\blacksquare}{\blacksquare}$.

52. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{c+d} = \frac{\blacksquare}{\blacksquare}$.

53. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+c}{b+d} = \frac{\blacksquare}{\blacksquare}$.

54. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+2b}{b} = \frac{\blacksquare}{\blacksquare}$.

Challenge

Algebra **Use properties of equality to justify each property of proportions.**

55. $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{b}{a} = \frac{d}{c}$.

56. $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a}{c} = \frac{b}{d}$.

57. $\frac{a}{b} = \frac{c}{d}$ is equivalent to $\frac{a+b}{b} = \frac{c+d}{d}$.

Algebra **Solve each proportion for the variable(s).**

58. $\frac{x-3}{3} = \frac{2}{x+2}$

59. $\frac{3-4x}{1+5x} = \frac{1}{2+3x}$

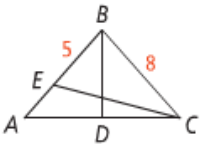
60. $\frac{x}{6} = \frac{x+10}{18} = \frac{4x}{y}$

Standardized Test Prep

GRIDDED RESPONSE

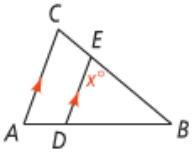
SAT/ACT

61. In the diagram at the right, $\overline{BD}$ is the perpendicular bisector of $\overline{AC}$. What is the length of $\overline{EA}$?



62. The measures of the angles of a triangle are in the extended ratio $6 : 3 : 1$. What is the measure of the largest angle?

63. In the diagram at the right, $\triangle ABC$ is an isosceles triangle with base $\overline{AC}$ and $m\angle ABC = 40$. What is the value of x?



64. In $\triangle PQR$, $PQ = 10$ in. and $QR = 12$ in. RP must be less than x inches. What is the value of x?

65. What is the length of the line segment joining $P(-3, -2)$ and $Q(9, 3)$?

Mixed Review

66. Write a coordinate proof of this statement: The diagonals of a square are perpendicular.

See Lesson 6-9.

In each exercise, identify two statements that contradict each other.

See Lessons 1-5 and 5-5.

67. I. $\triangle PQR$ is isosceles.
II. $\triangle PQR$ is an obtuse triangle.
III. $\triangle PQR$ is scalene.

68. I. $\angle 1 \cong \angle 2$
II. $\angle 1$ and $\angle 2$ are complementary.
III. $m\angle 1 + m\angle 2 = 180$

Get Ready! **To prepare for Lesson 7-2, do Exercise 69.**

69. $\triangle ABC \cong \triangle HIJ$. Name three pairs of congruent angles and three pairs of congruent sides.

See Lesson 4-1.

Algebra Review
Use With Lesson 7-2

Solving Quadratic Equations

Equations in the form $ax^2 + bx + c = 0$, where $a \neq 0$, are quadratic equations in standard form. You can solve some quadratic equations in standard form by factoring and using the Zero-Product Property:

If $ab = 0$, then $a = 0$ or $b = 0$.

You can solve all quadratic equations in standard form using the quadratic formula:

If $ax^2 + bx + c = 0$, where $a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example

Algebra **Solve for x. For irrational solutions, give both the exact answer and the answer rounded to the nearest hundredth.**

A $7x^2 + 6x - 1 = 0$ — The equation is in standard form.

$(7x - 1)(x + 1) = 0$ — Factor.

$7x - 1 = 0$ or $x + 1 = 0$ — Use the Zero-Product Property.

$x = \frac{1}{7}$ or $x = -1$ — Solve for x.

B $-3x^2 - 5x + 1 = 0$ — The equation is in standard form.

$a = -3, b = -5, c = 1$ — Identify a, b, and c.

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(-3)(1)}}{2(-3)}$ — Substitute in the quadratic formula.

$x = \dfrac{5 \pm \sqrt{37}}{-6}$ — Simplify.

$x = -\dfrac{5 + \sqrt{37}}{6}$ or $x = -\dfrac{5 - \sqrt{37}}{6}$ — Write the two solutions separately.

$x \approx -1.85$ or $x \approx 0.18$ — Use a calculator and round to the nearest hundredth.

Exercises

Algebra **Solve for x. For irrational solutions, give both the exact answer and the answer rounded to the nearest hundredth.**

1. $x^2 + 5x - 14 = 0$
2. $4x^2 - 13x + 3 = 0$
3. $2x^2 + 7x + 3 = 0$
4. $5x^2 + 2x - 2 = 0$
5. $2x^2 - 10x + 11 = 0$
6. $8x^2 - 2x - 3 = 0$
7. $2x^2 + 3x - 20 = 0$
8. $x^2 - x - 210 = 0$
9. $x^2 - 4x = 0$
10. $x^2 - 25 = 0$
11. $6x^2 + 10x = 5$
12. $1 = 2x^2 - 6x$

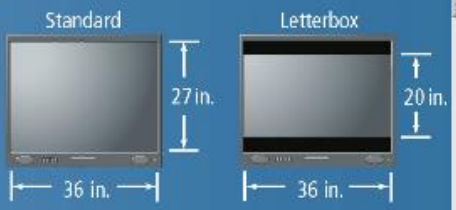
7-2 Similar Polygons

Objective To identify and apply similar polygons

You learned about ratios in the last lesson. Can you use ratios to help you solve the problem?

Getting Ready!

A movie theater screen is in the shape of a rectangle 45 ft wide by 25 ft high. Which of the TV screen formats at the right do you think would show the most complete scene from a movie shown on the theater screen? Explain.



Dynamic Activity
Similar Polygons

Lesson Vocabulary
- similar figures
- similar polygons
- extended proportion
- scale factor
- scale drawing
- scale

Similar figures have the same shape but not necessarily the same size. You can abbreviate *is similar to* with the symbol $\sim$.

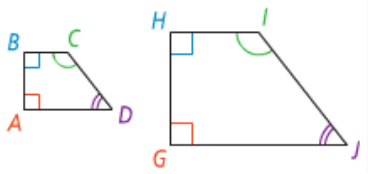
Essential Understanding You can use ratios and proportions to decide whether two polygons are similar and to find unknown side lengths of similar figures.

take note

Key Concept Similar Polygons

Define
Two polygons are **similar polygons** if corresponding angles are congruent and if the lengths of corresponding sides are proportional.

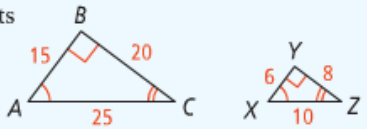
Diagram
$ABCD \sim GHIJ$



Symbols
$\angle A \cong \angle G$
$\angle B \cong \angle H$
$\angle C \cong \angle I$
$\angle D \cong \angle J$
$\frac{AB}{GH} = \frac{BC}{HI} = \frac{CD}{IJ} = \frac{AD}{GJ}$

You write a similarity statement with corresponding vertices in order, just as you write a congruence statement. When three or more ratios are equal, you can write an **extended proportion.** The proportion $\frac{AB}{GH} = \frac{BC}{HI} = \frac{CD}{IJ} = \frac{AD}{GJ}$ is an extended proportion.

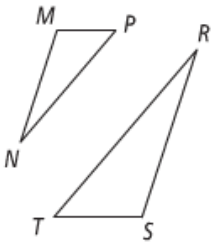
A **scale factor** is the ratio of corresponding linear measurements of two similar figures. The ratio of the lengths of corresponding sides $\overline{BC}$ and $\overline{YZ}$, or more simply stated, the ratio of corresponding sides, is $\frac{BC}{YZ} = \frac{20}{8} = \frac{5}{2}$. So the scale factor of $\triangle ABC$ to $\triangle XYZ$ is $\frac{5}{2}$ or 5 : 2.



$\triangle ABC \sim \triangle XYZ$

Problem 1 Understanding Similarity

$\triangle MNP \sim \triangle SRT$



A **What are the pairs of congruent angles?**

$\angle M \cong \angle S$, $\angle N \cong \angle R$, and $\angle P \cong \angle T$

B **What is the extended proportion for the ratios of corresponding sides?**

$\frac{MN}{SR} = \frac{NP}{RT} = \frac{MP}{ST}$

Think

How can you use the similarity statement to write ratios of corresponding sides?
Use the order of the sides in the similarity statement. $\overline{MN}$ corresponds to $\overline{SR}$, so $\frac{MN}{SR}$ is a ratio of corresponding sides.

Got It? **1.** $DEFG \sim HJKL$.

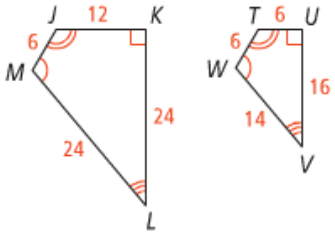
a. What are the pairs of congruent angles?

b. What is the extended proportion for the ratios of the lengths of corresponding sides?

Problem 2 Determining Similarity

Are the polygons similar? If they are, write a similarity statement and give the scale factor.

A ***JKLM* and *TUVW***



Step 1 Identify pairs of congruent angles.

$\angle J \cong \angle T$, $\angle K \cong \angle U$, $\angle L \cong \angle V$, and $\angle M \cong \angle W$

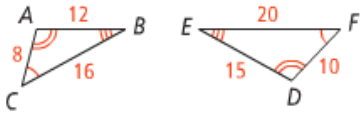
Step 2 Compare the ratios of corresponding sides.

$\frac{JK}{TU} = \frac{12}{6} = \frac{2}{1}$ $\quad \frac{KL}{UV} = \frac{24}{16} = \frac{3}{2}$

$\frac{LM}{VW} = \frac{24}{14} = \frac{12}{7}$ $\quad \frac{JM}{TW} = \frac{6}{6} = \frac{1}{1}$

Corresponding sides are not proportional, so the polygons are not similar.

B $\triangle ABC$ **and** $\triangle EFD$



Step 1 Identify pairs of congruent angles.

$\angle A \cong \angle D$, $\angle B \cong \angle E$, and $\angle C \cong \angle F$

Step 2 Compare the ratios of corresponding sides.

$\frac{AB}{DE} = \frac{12}{15} = \frac{4}{5}$ $\quad \frac{BC}{EF} = \frac{16}{20} = \frac{4}{5}$ $\quad \frac{AC}{DF} = \frac{8}{10} = \frac{4}{5}$

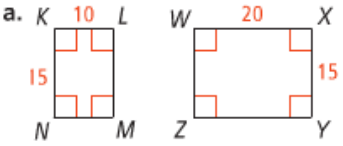
Yes; $\triangle ABC \sim \triangle DEF$ and the scale factor is $\frac{4}{5}$ or $4 : 5$.

Think

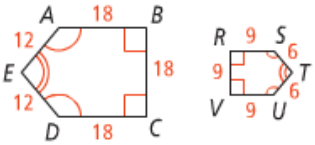
How do you identify corresponding sides?
The included side between a pair of angles of one polygon corresponds to the included side between the corresponding pair of congruent angles of another polygon.

Got It? **2.** Are the polygons similar? If they are, write a similarity statement and give the scale factor.

a.



b.



Problem 3 Using Similar Polygons

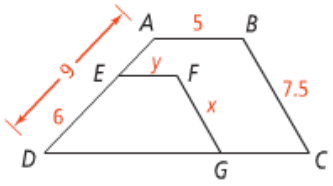
Plan

Can you rely on the diagram alone to set up the proportion? No, you need to use the similarity statement to identify corresponding sides in order to write ratios that are equal.

Algebra $ABCD \sim EFGD$. **What is the value of x?**

(A) 4.5 (C) 7.2

(B) 5 (D) 11.25



$\frac{FG}{BC} = \frac{ED}{AD}$	Corresponding sides of similar polygons are proportional.
$\frac{x}{7.5} = \frac{6}{9}$	Substitute.
$9x = 45$	Cross Products Property
$x = 5$	Divide each side by 9.

The value of x is 5. The correct answer is B.

Got It? **3.** Use the diagram in Problem 3. What is the value of y?

Problem 4 Using Similarity

Think

You can't solve the problem until you know which dimension fills the space first.

Design **Your class is making a rectangular poster for a rally. The poster's design is 6 in. high by 10 in. wide. The space allowed for the poster is 4 ft high by 8 ft wide. What are the dimensions of the largest poster that will fit in the space?**

Step 1 Determine whether the height or width will fill the space first.

Height: $4 \text{ ft} = 48 \text{ in.}$ Width: $8 \text{ ft} = 96 \text{ in.}$

$48 \text{ in.} \div 6 \text{ in.} = 8$ $96 \text{ in.} \div 10 \text{ in.} = 9.6$

The design can be enlarged at most 8 times.

Step 2 The greatest height is 48 in., so find the width.

$\frac{6}{48} = \frac{10}{x}$	Corresponding sides of similar polygons are proportional.
$6x = 480$	Cross Products Property
$x = 80$	Divide each side by 6.

The largest poster is 48 in. by 80 in. or 4 ft by $6\frac{2}{3}$ ft.

Got It? **4.** Use the same poster design in Problem 4. What are the dimensions of the largest complete poster that will fit in a space 3 ft high by 4 ft wide?

In a **scale drawing,** all lengths are proportional to their corresponding actual lengths. The **scale** is the ratio that compares each length in the scale drawing to the actual length. The lengths used in a scale can be in different units. For example, a scale might be written as 1 cm to 50 km, 1 in. = 100 mi, or 1 in. : 10 ft.

You can use proportions to find the actual dimensions represented in a scale drawing.

Problem 5 Using a Scale Drawing

Design **The diagram shows a scale drawing of the Golden Gate Bridge in San Francisco. The distance between the two towers is the main span. What is the actual length of the main span of the bridge?**

Think

Why is it helpful to use a scale in different units?
1 cm : 200 m in the same units would be 1 cm : 20,000 cm. When solving the problem, $\frac{1}{200}$ is easier to work with than $\frac{1}{20{,}000}$.

The length of the main span in the scale drawing is 6.4 cm. Let s represent the main span of the bridge. Use the scale to set up a proportion.

$\frac{1}{200} = \frac{6.4}{s}$ $\quad \frac{\text{length in drawing (cm)}}{\text{actual length (m)}}$

$s = 1280$ Cross Products Property

The actual length of the main span of the bridge is 1280 m.

Got It? **5. a.** Use the scale drawing in Problem 5. What is the actual height of the towers above the roadway?

b. Reasoning The Space Needle in Seattle is 605 ft tall. A classmate wants to make a scale drawing of the Space Needle on an $8\frac{1}{2}$ in.-by-11 in. sheet of paper. He decides to use the scale 1 in. = 50 ft. Is this a reasonable scale? Explain.

Lesson Check

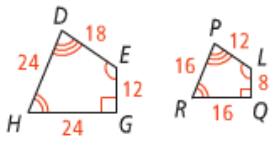
Do you know HOW?

JDRT ~ *WHYX*. Complete each statement.

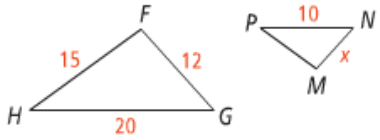
1. $\angle D \cong$?

2. $\frac{RT}{YX} = \frac{\blacksquare}{WX}$

3. Are the polygons similar? If they are, write a similarity statement and give the scale factor.



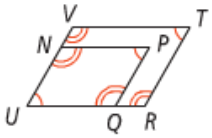
4. $\triangle FGH \sim \triangle MNP$. What is the value of x?



Do you UNDERSTAND?

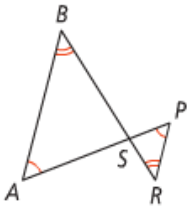
5. **Vocabulary** What does the scale on a scale drawing indicate?

6. **Error Analysis** The polygons at the right are similar. Which similarity statement is *not* correct? Explain.
   - **A.** *TRUV* ~ *NPQU*
   - **B.** *RUVT* ~ *QUNP*



7. **Reasoning** Is similarity reflexive? Transitive? Symmetric? Justify your reasoning.

8. The triangles at the right are similar. What are three similarity statements for the triangles?

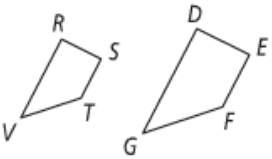


Practice and Problem-Solving Exercises

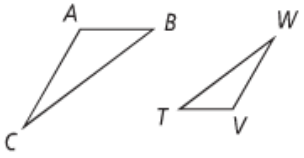
A Practice

List the pairs of congruent angles and the extended proportion that relates the corresponding sides for the similar polygons. See Problem 1.

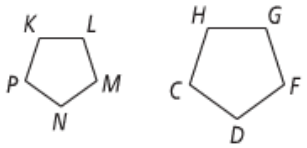
9. *RSTV* ~ *DEFG*



10. $\triangle CAB \sim \triangle WVT$

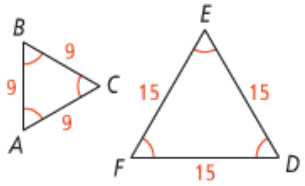


11. *KLMNP* ~ *HGFDC*

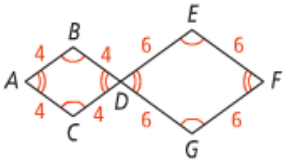


Determine whether the polygons are similar. If so, write a similarity statement and give the scale factor. If not, explain. See Problem 2.

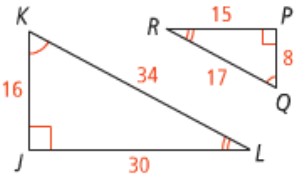
12.

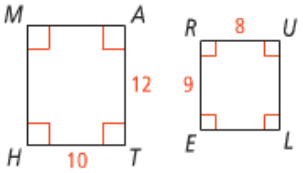


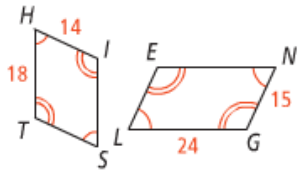
13.

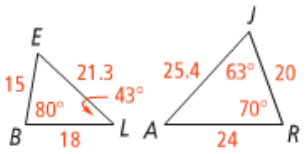


14.

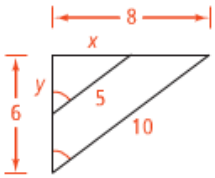


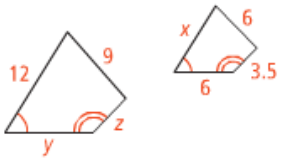
15. 

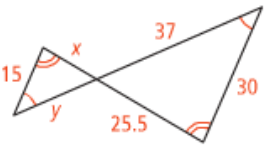
16. 

17. 

Algebra The polygons are similar. Find the value of each variable. See Problem 3.

18. 

19. 

20. 

21. Web Page Design The space allowed for the mascot on a school's Web page is 120 pixels wide by 90 pixels high. Its digital image is 500 pixels wide by 375 pixels high. What is the largest image of the mascot that will fit on the Web page? See Problem 4.

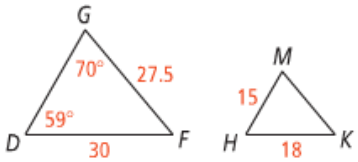
22. Art The design for a mural is 16 in. wide and 9 in. high. What are the dimensions of the largest possible complete mural that can be painted on a wall 24 ft wide by 14 ft high?

23. Architecture You want to make a scale drawing of New York City's Empire State Building using the scale 1 in. = 250 ft. If the building is 1250 ft tall, how tall should you make the building in your scale drawing? See Problem 5.

24. Cartography A cartographer is making a map of Pennsylvania. She uses the scale 1 in. = 10 mi. The actual distance between Harrisburg and Philadelphia is about 95 mi. How far apart should she place the two cities on the map?

B Apply

In the diagram below, $\triangle DFG \sim \triangle HKM$. Find each of the following.



25. the scale factor of $\triangle HKM$ to $\triangle DFG$

26. $m\angle K$

27. $\frac{GD}{MH}$

28. MK

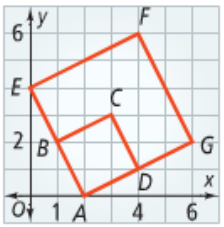
29. GD

30. Flags A company produces a standard-size U.S. flag that is 3 ft by 5 ft. The company also produces a giant-size flag that is similar to the standard-size flag. If the shorter side of the giant-size flag is 36 ft, what is the length of its longer side?

31. a. Coordinate Geometry What are the measures of $\angle A$, $\angle ABC$, $\angle BCD$, $\angle CDA$, $\angle E$, $\angle F$, and $\angle G$? Explain.

b. What are the lengths of $\overline{AB}$, $\overline{BC}$, $\overline{CD}$, $\overline{DA}$, $\overline{AE}$, $\overline{EF}$, $\overline{FG}$, and $\overline{AG}$?

c. Is $ABCD$ similar to $AEFG$? Justify your answer.



32. Think About a Plan The Davis family is planning to drive from San Antonio to Houston. About how far will they have to drive?

- How can you find the distance between the two cities on the map?
- What proportion can you set up to solve the problem?

33. Reasoning Two polygons have corresponding side lengths that are proportional. Can you conclude that the polygons are similar? Justify your reasoning.

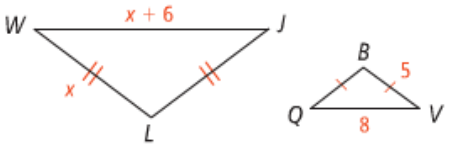
34. Writing Explain why two congruent figures must also be similar. Include scale factor in your explanation.

35. $\triangle JLK$ and $\triangle RTS$ are similar. The scale factor of $\triangle JLK$ to $\triangle RTS$ is $3 : 1$. What is the scale factor of $\triangle RTS$ to $\triangle JLK$?

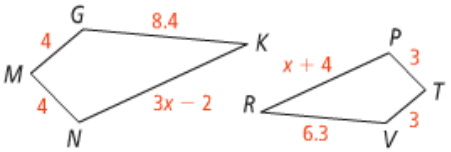
36. Open-Ended Draw and label two different similar quadrilaterals. Write a similarity statement for each and give the scale factor.

Algebra Find the value of x. Give the scale factor of the polygons.

37. $\triangle WLJ \sim \triangle QBV$



38. $GKNM \sim VRPT$



Sports Choose a scale and make a scale drawing of each rectangular playing surface.

39. A soccer field is 110 yd by 60 yd.

40. A volleyball court is 60 ft by 30 ft.

41. A tennis court is 78 ft by 36 ft.

42. A football field is 360 ft by 160 ft.

Determine whether each statement is *always*, *sometimes*, or *never* true.

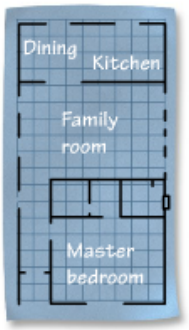
43. Any two regular pentagons are similar.

44. A hexagon and a triangle are similar.

45. A square and a rhombus are similar.

46. Two similar rectangles are congruent.

47. Architecture The scale drawing at the right is part of a floor plan for a home. The scale is 1 cm = 10 ft. What are the actual dimensions of the family room?



Challenge

48. The lengths of the sides of a triangle are in the extended ratio $2 : 3 : 4$. The perimeter of the triangle is 54 in.

a. The length of the shortest side of a similar triangle is 16 in. What are the lengths of the other two sides of this triangle?

b. Compare the ratio of the perimeters of the two triangles to their scale factor. What do you notice?

49. In rectangle $BCEG$, $BC : CE = 2 : 3$. In rectangle $LJAW$, $LJ : JA = 2 : 3$. Show that $BCEG \sim LJAW$.

50. Prove the following statement: If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle GHK$, then $\triangle ABC \sim \triangle GHK$.

Standardized Test Prep

SAT/ACT

51. $PQRS \sim JKLM$ with a scale factor of $4 : 3$. $QR = 8$ cm. What is the value of KL?

(A) 6 cm (B) 8 cm (C) $10\frac{2}{3}$ cm (D) 24 cm

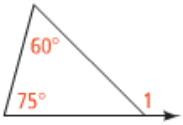
52. Which of the following is NOT a property of an isosceles trapezoid?

(F) The base angles are congruent. (H) The diagonals are perpendicular.

(G) The legs are congruent. (I) The diagonals are congruent.

53. In the diagram at the right, what is $m\angle 1$?

(A) 45 (B) 75 (C) 125 (D) 135



Short Response

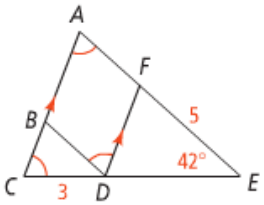
54. A high school community-action club plans to build a circular play area in a city park. The club members need to buy materials to enclose the area and sand to fill the area. For a 9-ft-diameter play area, what will be the circumference and area rounded to the nearest hundredth?

Mixed Review

If $\frac{x}{7} = \frac{y}{9}$, complete each statement using the properties of proportions. See Lesson 7-1.

55. $9x =$ ■ **56.** $\frac{x}{y} =$ ■/■ **57.** $\frac{x+7}{7} =$ ■/■

Use the diagram for Exercises 58–61. See Lesson 4-5.



58. Name the isosceles triangles in the figure.

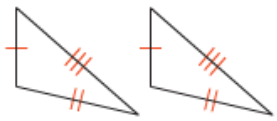
59. $\overline{CD} \cong$? $\cong$?

60. $AE =$? **61.** $m\angle A =$?

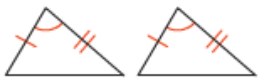
Get Ready! **To prepare for Lesson 7-3, do Exercises 62–64.**

How can you prove that the triangles are congruent? See Lessons 4-2 and 4-3.

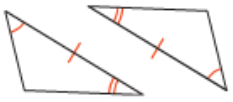
62.



63.



64.



Concept Byte

Use With Lesson 7-2

EXTENSION

Fractals

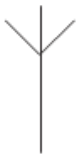
Fractals are objects that have three important properties:

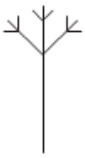
- You can form fractals by repeating steps. This process is called *iteration*.
- They require infinitely many iterations. In practice, you can continue until the objects become too small to draw. Even then the steps could continue in your mind.
- At each stage, a portion of the object is a reduced copy of the entire object at the previous stage. This property is called *self-similarity*.

Example 1

The segment at the right of length 1 unit is Stage 0 of a fractal tree. Draw Stage 1 and Stage 2 of the tree. For each stage, draw two branches from the top third of each segment.

- To draw Stage 1, find the point that is $\frac{1}{3}$ unit from the top of the segment. From this point, draw two segments of length $\frac{1}{3}$ unit.
- To draw Stage 2, find the point that is $\frac{1}{3}$ unit from the top of each branch of Stage 1. From each of these points, draw two segments of length $\frac{1}{9}$ unit.





Amazingly, some fractals are used to describe natural formations such as mountain ranges and clouds. In 1904, Swedish mathematician Helge von Koch made the Koch Curve, a fractal that is used to model coastlines.

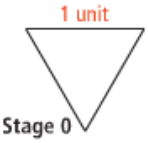
Example 2

The segment at the right of length 1 unit is Stage 0 of a Koch Curve. Draw Stages 1–4 of the curve. For each stage, replace the middle third of each segment with two segments, both equal in length to the middle third.

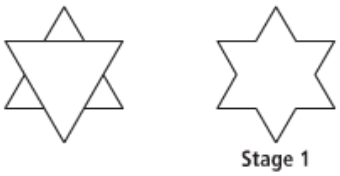
- For Stage 1, replace the middle third with two segments that are each $\frac{1}{3}$ unit long.
- For Stage 2, replace the middle third of each segment of Stage 1 with two segments that are each $\frac{1}{9}$ unit long.
- Continue with a third and fourth iteration.

Example 3

The equilateral triangle at the right is Stage 0 of a Koch Snowflake. Draw Stage 1 of the snowflake by first drawing an equilateral triangle on the middle third of each side. Then erase the middle third of each side of the original triangle.



- To draw an equilateral triangle on the middle third of a side, find the two points that are $\frac{1}{3}$ unit from an endpoint of the side. From each point, draw a segment of length $\frac{1}{3}$ unit. Each segment must make a 60° angle with the side of the original triangle.



Exercises

1. Draw Stage 3 of the fractal tree in Example 1.

Use the Koch Curve in Example 2 for Exercises 2–4.

2. Complete the table to find the length of the Koch Curve at each stage.

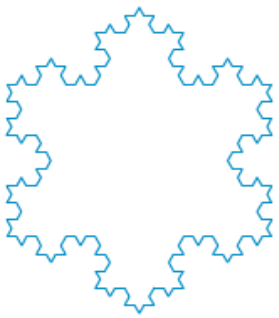
Stage	0	1	2
Length	1	■	■

3. Examine the results of Exercise 2 and look for a pattern. Use this pattern to predict the length of the Koch Curve at Stage 3 and at Stage 4.

4. Suppose you are able to complete a Koch Curve to Stage n.
   a. Write an expression for the length of the curve.
   b. What happens to the length of the curve as n increases?

5. Draw Stage 2 of the Koch Snowflake in Example 3.

Stage 3 of the Koch Snowflake is shown at the right. Use it and the earlier stages to answer Exercises 6–8.



6. At each stage, is the snowflake equilateral?

7. a. Complete the table to find the perimeter at each stage.

Stage	Number of Sides	Length of a Side	Perimeter
0	3	1	3
1	■	$\frac{1}{3}$	■
2	48	■	■
3	■	■	■

   b. Predict the perimeter at Stage 4.
   c. Will there be a stage at which the perimeter is greater than 100 units? Explain.

8. Exercises 4 and 7 suggest that there is no bound on the perimeter of the Koch Snowflake. Is this true about the area of the Koch Snowflake? Explain.

7-3 Proving Triangles Similar

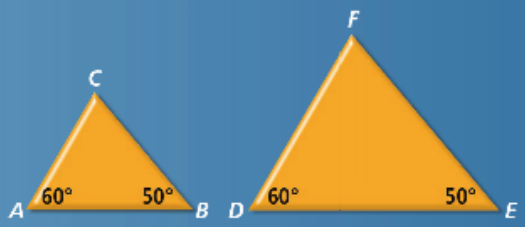
Objectives To use the AA ~ Postulate and the SAS ~ and SSS ~ Theorems
To use similarity to find indirect measurements

You've already learned how to decide whether two polygons are similar. Use the same strategy here.

SOLVE IT!

Getting Ready!

Are the triangles similar? How do you know? (Hint: Use a centimeter ruler to measure the sides of each triangle.)



VOCABULARY

Lesson Vocabulary
- indirect measurement

In the Solve It, you determined whether the two triangles are similar. That is, you needed information about all three pairs of angles and all three pairs of sides. In this lesson, you'll learn an easier way to determine whether two triangles are similar.

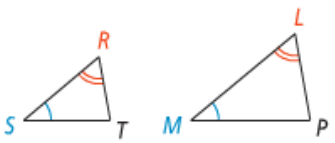
Essential Understanding You can show that two triangles are similar when you know the relationships between only two or three pairs of corresponding parts.

take note

Postulate 7-1 Angle-Angle Similarity (AA ~) Postulate

Postulate
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.

If . . .
$\angle S \cong \angle M$ and $\angle R \cong \angle L$



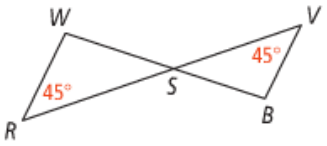
Then . . .
$\triangle SRT \sim \triangle MLP$

Plan

What do you need to show that the triangles are similar?
To use the AA ~ Postulate, you need to prove that two pairs of angles are congruent.

Problem 1 Using the AA ~ Postulate

Are the two triangles similar? How do you know?

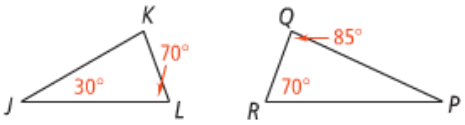


A $\triangle RSW$ and $\triangle VSB$

$\angle R \cong \angle V$ because both angles measure 45°.
$\angle RSW \cong \angle VSB$ because vertical angles are congruent.
So, $\triangle RSW \sim \triangle VSB$ by the AA ~ Postulate.

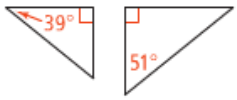
B $\triangle JKL$ and $\triangle PQR$

$\angle L \cong \angle R$ because both angles measure 70°.
By the Triangle Angle-Sum Theorem,
$m\angle K = 180 - 30 - 70 = 80$ and
$m\angle P = 180 - 85 - 70 = 25$. Only one pair of angles is congruent. So, $\triangle JKL$ and $\triangle PQR$ are *not* similar.

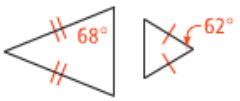


Got It? **1.** Are the two triangles similar? How do you know?

a.



b.



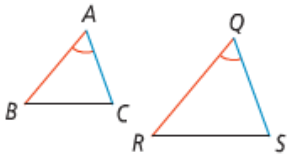
Here are two other ways to determine whether two triangles are similar.

take note

Theorem 7-1 Side-Angle-Side Similarity (SAS ~) Theorem

Theorem
If an angle of one triangle is congruent to an angle of a second triangle, and the sides that include the two angles are proportional, then the triangles are similar.

If . . .
$\frac{AB}{QR} = \frac{AC}{QS}$ and $\angle A \cong \angle Q$



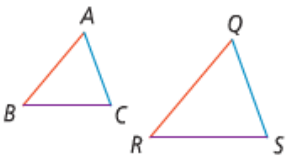
Then . . .
$\triangle ABC \sim \triangle QRS$

You will prove Theorem 7-1 in Exercise 35.

Theorem 7-2 Side-Side-Side Similarity (SSS ~) Theorem

Theorem
If the corresponding sides of two triangles are proportional, then the triangles are similar.

If . . .
$\frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS}$



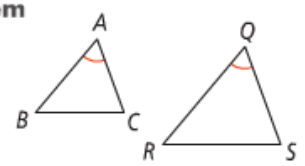
Then . . .
$\triangle ABC \sim \triangle QRS$

You will prove Theorem 7-2 in Exercise 36.

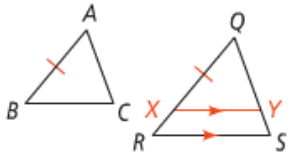
Proof **Proof of Theorem 7-1: Side-Angle-Side Similarity Theorem**

Given: $\frac{AB}{QR} = \frac{AC}{QS}$, $\angle A \cong \angle Q$

Prove: $\triangle ABC \sim \triangle QRS$



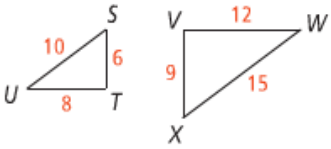
Plan for Proof: Choose X on $\overline{RQ}$ so that $QX = AB$. Draw $\overleftrightarrow{XY} \parallel \overline{RS}$. Show that $\triangle QXY \sim \triangle QRS$ by the AA $\sim$ Postulate. Then use the proportion $\frac{QX}{QR} = \frac{QY}{QS}$ and the given proportion $\frac{AB}{QR} = \frac{AC}{QS}$ to show that $AC = QY$. Then prove that $\triangle ABC \cong \triangle QXY$. Finally, prove that $\triangle ABC \sim \triangle QRS$ by the AA $\sim$ Postulate.



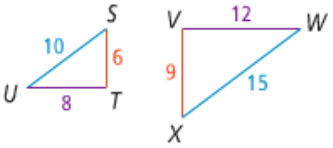
Problem 2 Verifying Triangle Similarity

Are the triangles similar? If so, write a similarity statement for the triangles.

A



Use the side lengths to identify corresponding sides. Then set up ratios for each pair of corresponding sides.



Shortest sides $\frac{ST}{XV} = \frac{6}{9} = \frac{2}{3}$

Longest sides $\frac{US}{WX} = \frac{10}{15} = \frac{2}{3}$

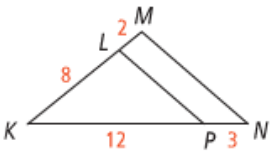
Remaining sides $\frac{TU}{VW} = \frac{8}{12} = \frac{2}{3}$

All three ratios are equal, so corresponding sides are proportional. $\triangle STU \sim \triangle XVW$ by the SSS $\sim$ Theorem.

Plan

How can you make it easier to identify corresponding sides and angles?
Sketch and label two separate triangles.

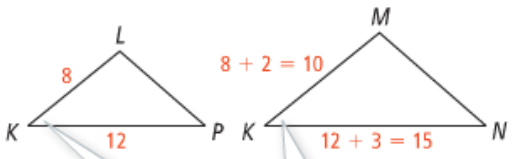
B



$\angle K \cong \angle K$ by the Reflexive Property of Congruence.

$\frac{KL}{KM} = \frac{8}{10} = \frac{4}{5}$ and $\frac{KP}{KN} = \frac{12}{15} = \frac{4}{5}$.

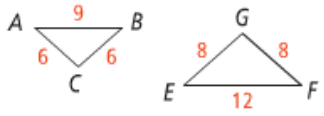
So, $\triangle KLP \sim \triangle KMN$ by the SAS $\sim$ Theorem.



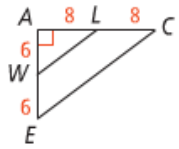
$\angle K$ is the *included* angle between two known sides in each triangle.

Got It? **2.** Are the triangles similar? If so, write a similarity statement for the triangles and explain how you know the triangles are similar.

a.



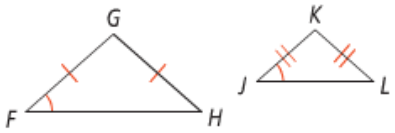
b.



Proof **Problem 3** **Proving Triangles Similar**

Given: $\overline{FG} \cong \overline{GH}$, $\overline{JK} \cong \overline{KL}$, $\angle F \cong \angle J$

Prove: $\triangle FGH \sim \triangle JKL$



Know
The triangles are isosceles, so the base angles are congruent.

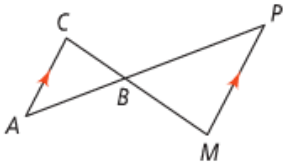
Need
You need to show that the triangles are similar.

Plan
Find two pairs of corresponding congruent angles and use the AA ~ Postulate to prove the triangles are similar.

Statements	Reasons
1) $\overline{FG} \cong \overline{GH}$, $\overline{JK} \cong \overline{KL}$	1) Given
2) $\triangle FGH$ is isosceles. $\triangle JKL$ is isosceles.	2) Def. of an isosceles $\triangle$
3) $\angle F \cong \angle H$, $\angle J \cong \angle L$	3) Base $\measuredangle$ of an isosceles $\triangle$ are $\cong$.
4) $\angle F \cong \angle J$	4) Given
5) $\angle H \cong \angle J$	5) Transitive Property of $\cong$
6) $\angle H \cong \angle L$	6) Transitive Property of $\cong$
7) $\triangle FGH \sim \triangle JKL$	7) AA ~ Postulate

Got It? **3. a. Given:** $\overline{MP} \parallel \overline{AC}$

Prove: $\triangle ABC \sim \triangle PBM$



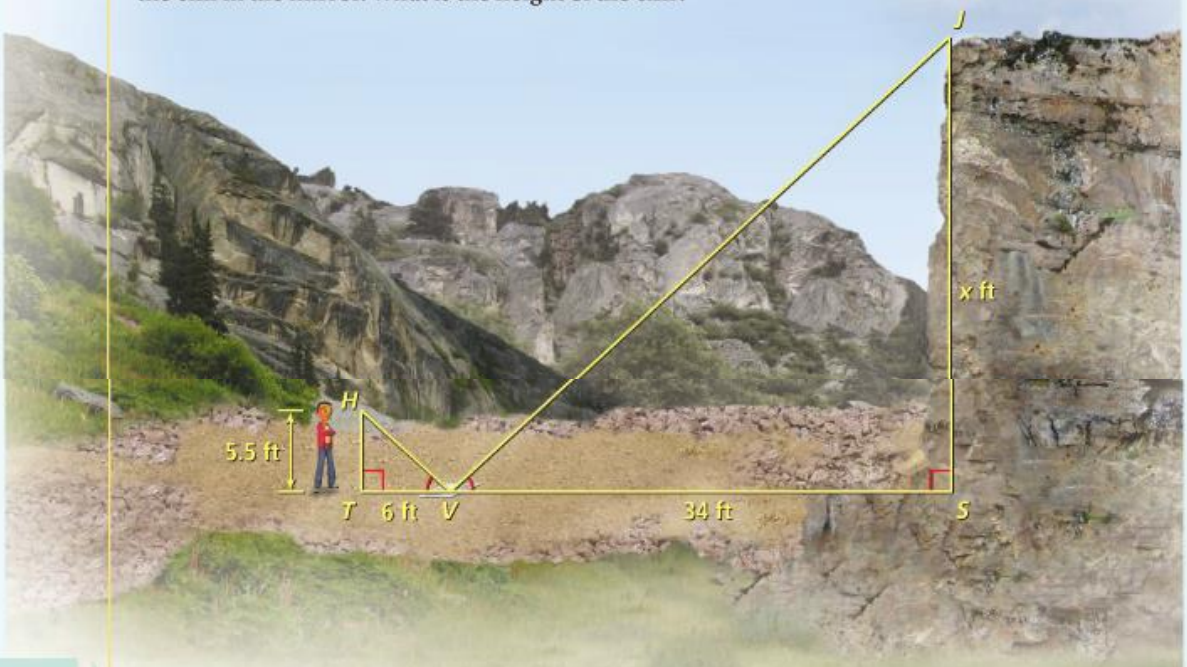
b. Reasoning For the figure at the right, suppose you are given only that $\frac{CA}{PM} = \frac{CB}{MB}$. Could you prove that the triangles are similar? Explain.

Essential Understanding Sometimes you can use similar triangles to find lengths that cannot be measured easily using a ruler or other measuring device.

You can use **indirect measurement** to find lengths that are difficult to measure directly. One method of indirect measurement uses the fact that light reflects off a mirror at the same angle at which it hits the mirror.

Problem 4 Finding Lengths in Similar Triangles

Rock Climbing **Before rock climbing, Darius wants to know how high he will climb. He places a mirror on the ground and walks backward until he can see the top of the cliff in the mirror. What is the height of the cliff?**



Plan

Before solving for x, verify that the triangles are similar. $\triangle HTV \sim \triangle JSV$ by the AA $\sim$ Postulate because $\angle T \cong \angle S$ and $\angle HVT \cong \angle JVS$.

$\triangle HTV \sim \triangle JSV$	AA $\sim$ Postulate
$\frac{HT}{JS} = \frac{TV}{SV}$	Corresponding sides of $\sim$ triangles are proportional.
$\frac{5.5}{x} = \frac{6}{34}$	Substitute.
$187 = 6x$	Cross Products Property
$31.2 \approx x$	Solve for x.

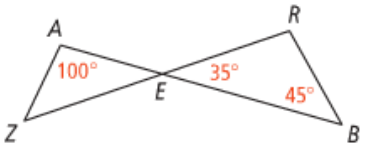
The cliff is about 31 ft high.

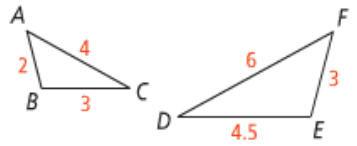
Got It? **4. Reasoning** Why is it important that the ground be flat to use the method of indirect measurement illustrated in Problem 4? Explain.

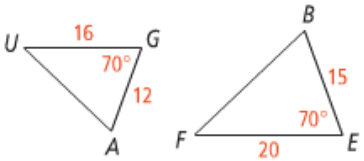
Lesson Check

Do you know HOW?

Are the triangles similar? If yes, write a similarity statement and explain how you know they are similar.

1. 

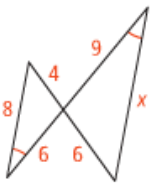
2. 

3. 

Do you UNDERSTAND?

4. **Vocabulary** How could you use indirect measurement to find the height of the flagpole at your school?

5. **Error Analysis** Which solution for the value of x in the figure at the right is *not* correct? Explain.



A.
$$\frac{4}{8} = \frac{8}{x}$$
$$4x = 72$$
$$x = 18$$

B.
$$\frac{8}{x} = \frac{4}{6}$$
$$48 = 4x$$
$$12 = x$$

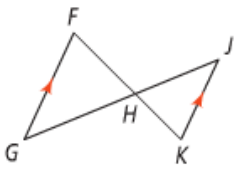
6. a. **Compare and Contrast** How are the SAS Similarity Theorem and the SAS Congruence Postulate alike? How are they different?
   b. How are the SSS Similarity Theorem and the SSS Congruence Postulate alike? How are they different?

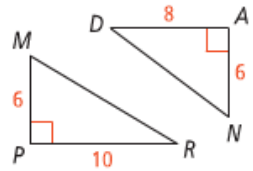
Practice and Problem-Solving Exercises

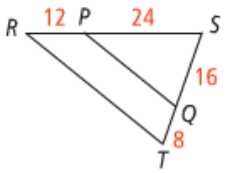
A Practice

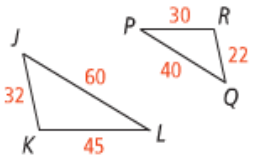
Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

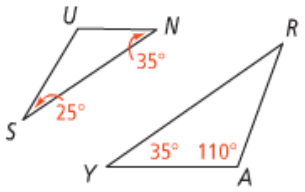
See Problems 1 and 2.

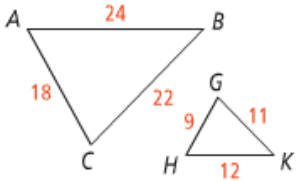
7. 

8. 

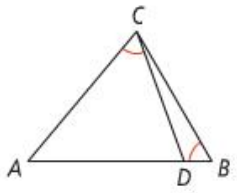
9. 

10. 

11. 

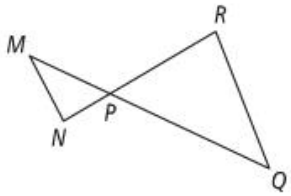
12. 

13. Proof **Given:** $\angle ABC \cong \angle ACD$
Prove: $\triangle ABC \sim \triangle ACD$



14. Proof **Given:** $PR = 2NP$, $PQ = 2MP$
Prove: $\triangle MNP \sim \triangle QRP$

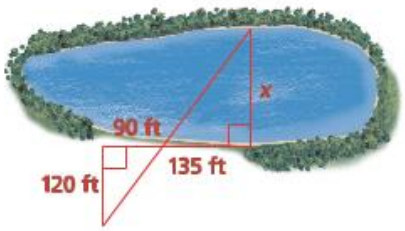
See Problem 3.



Indirect Measurement Explain why the triangles are similar. Then find the distance represented by x.

See Problem 4.

15.



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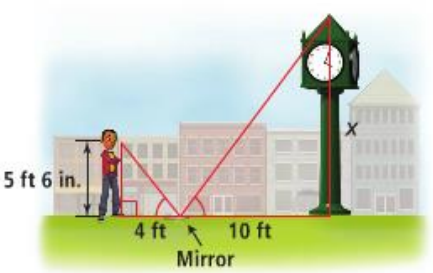
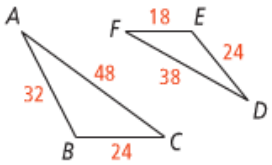


17. Washington Monument At a certain time of day, a 1.8-m-tall person standing next to the Washington Monument casts a 0.7-m shadow. At the same time, the Washington Monument casts a 65.8-m shadow. How tall is the Washington Monument?

B Apply

Can you conclude that the triangles are similar? If so, state the postulate or theorem you used and write a similarity statement. If not, explain.

18.



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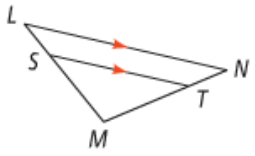

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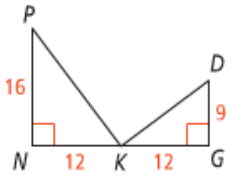


21. **a.** Are two isosceles triangles always similar? Explain.
b. Are two right isosceles triangles always similar? Explain.

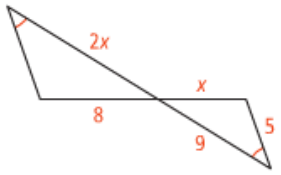
22. Think About a Plan On a sunny day, a classmate uses indirect measurement to find the height of a building. The building's shadow is 12 ft long and your classmate's shadow is 4 ft long. If your classmate is 5 ft tall, what is the height of the building?

- Can you draw and label a diagram to represent the situation?
- What proportion can you use to solve the problem?

23. Indirect Measurement A 2-ft vertical post casts a 16-in. shadow at the same time a nearby cell phone tower casts a 120-ft shadow. How tall is the cell phone tower?

Algebra For each pair of similar triangles, find the value of x.

24.



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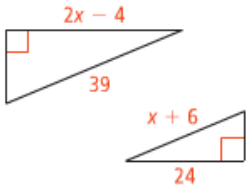

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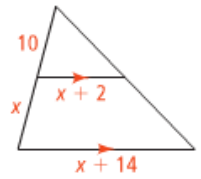
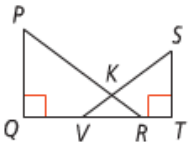
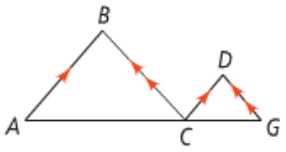


27. Proof **Given:** $\overline{PQ} \perp \overline{QT}, \overline{ST} \perp \overline{TQ}, \frac{PQ}{ST} = \frac{QR}{TV}$
Prove: $\triangle VKR$ is isosceles.



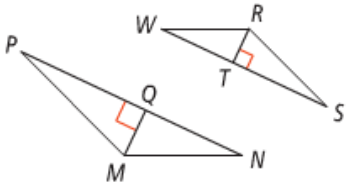
28. Proof **Given:** $\overline{AB} \parallel \overline{CD}, \overline{BC} \parallel \overline{DG}$
Prove: $AB \cdot CG = CD \cdot AC$



29. Reasoning Does any line that intersects two sides of a triangle and is parallel to the third side of the triangle form two similar triangles? Justify your reasoning.

30. Constructions Draw any $\triangle ABC$ with $m\angle C = 30$. Use a straightedge and compass to construct $\triangle LKJ$ so that $\triangle LKJ \sim \triangle ABC$.

31. Reasoning In the diagram at the right, $\triangle PMN \sim \triangle SRW$. $\overline{MQ}$ and $\overline{RT}$ are altitudes. The scale factor of $\triangle PMN$ to $\triangle SRW$ is 4 : 3. What is the ratio of $\overline{MQ}$ to $\overline{RT}$? Explain how you know.

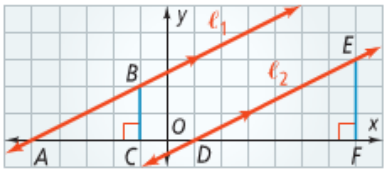


32. Coordinate Geometry Proof $\triangle ABC$ has vertices $A(0, 0)$, $B(2, 4)$, and $C(4, 2)$. $\triangle RST$ has vertices $R(0, 3)$, $S(-1, 5)$, and $T(-2, 4)$. Prove that $\triangle ABC \sim \triangle RST$. (*Hint:* Graph $\triangle ABC$ and $\triangle RST$ in the coordinate plane.)

Challenge

33. Proof Write a proof of the following: Any two nonvertical parallel lines have equal slopes.

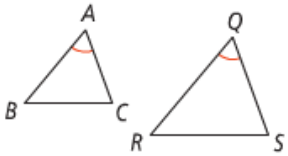
Given: Nonvertical lines ℓ_1 and ℓ_2, $\ell_1 \parallel \ell_2$, $\overline{EF}$ and $\overline{BC}$ are $\perp$ to the x-axis
Prove: $\frac{BC}{AC} = \frac{EF}{DF}$



34. Proof Use the diagram in Exercise 33. Prove: Any two nonvertical lines with equal slopes are parallel.

35. Proof Prove the Side-Angle-Side Similarity Theorem (Theorem 7-1).

Given: $\frac{AB}{QR} = \frac{AC}{QS}, \angle A \cong \angle Q$
Prove: $\triangle ABC \sim \triangle QRS$

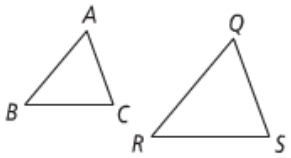


36. Prove the Side-Side-Side Similarity Theorem (Theorem 7-2).

Proof

Given: $\frac{AB}{QR} = \frac{AC}{QS} = \frac{BC}{RS}$

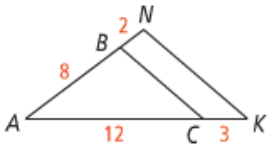
Prove: $\triangle ABC \sim \triangle QRS$



Standardized Test Prep

SAT/ACT

37. Complete the statement $\triangle ABC \sim$? . By which postulate or theorem are the triangles similar?



(A) $\triangle AKN$; SSS $\sim$ (C) $\triangle ANK$; SAS $\sim$

(B) $\triangle AKN$; SAS $\sim$ (D) $\triangle ANK$; AA $\sim$

38. $\angle 1$ and $\angle 2$ are alternate interior angles formed by two parallel lines and a transversal. If $m\angle 2 = 68$, what is $m\angle 1$?

(F) 22 (G) 68 (H) 112 (I) 122

39. The length of a rectangle is twice its width. If the perimeter of the rectangle is 72 in., what is the length of the rectangle?

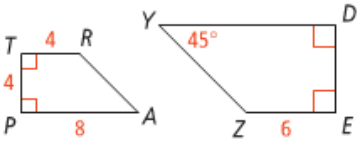
(A) 12 in. (B) 18 in. (C) 24 in. (D) 36 in.

Extended Response

40. Graph $A(2, 4)$, $B(4, 6)$, $C(6, 4)$, and $D(4, 2)$. What type of polygon is $ABCD$? Justify your answer.

Mixed Review

$TRAP \sim EZYD$. Use the diagram at the right to find the following.



See Lesson 7-2.

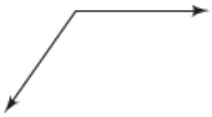
41. the scale factor of $TRAP$ to $EZYD$

42. $m\angle R$ **43.** DY **44.** $\frac{DE}{PT}$

Use a protractor to find the measure of each angle. Classify the angle as *acute*, *right*, *obtuse*, or *straight*.

See Lesson 1-4.

45.



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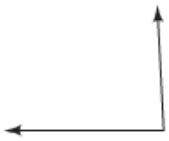


47.

48.

Get Ready! **To prepare for Lesson 7-4, do Exercises 49–52.**

Algebra **Identify the means and extremes of each proportion. Then solve for x.**

See Lesson 7-1.

49. $\frac{x}{8} = \frac{18}{24}$ **50.** $\frac{12}{m} = \frac{18}{20}$ **51.** $\frac{15}{x+2} = \frac{9}{x}$ **52.** $\frac{x-3}{x+4} = \frac{5}{9}$

7 Mid-Chapter Quiz

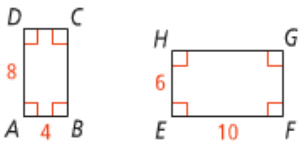
MathXL® for School
Go to PowerGeometry.com

Do you know HOW?

1. A bookcase is 4 ft tall. A model of the bookcase is 6 in. tall. What is the ratio of the height of the model bookcase to the height of the real bookcase?

2. If $\frac{a}{b} = \frac{9}{10}$, complete this statement: $\frac{a}{9} = \frac{\blacksquare}{\blacksquare}$.

3. Are the two polygons shown below similar? If so, give the similarity ratio of the first polygon to the second. If not, explain.



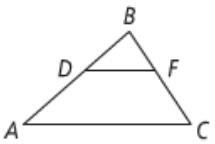
Solve each proportion.

4. $\frac{y}{6} = \frac{18}{54}$

5. $\frac{5}{7} = \frac{x-2}{4}$

6. On the scale drawing of a floor plan 2 in. = 5 ft. A room is 7 in. long on the scale drawing. Find the actual length of the room.

$\triangle ABC \sim \triangle DBF$. Complete each statement.

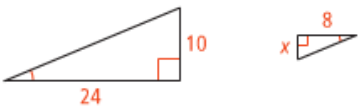


7. $m\angle A = m\angle _?_$

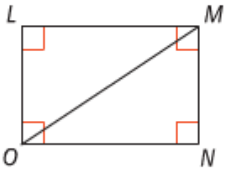
8. $\frac{AB}{DB} = \frac{BC}{\blacksquare}$

9. A postcard is 6 in. by 4 in. A printing shop will enlarge it so that the longer side is any length up to 3 ft. Find the dimensions of the biggest enlargement.

10. **Algebra** Find the value of x.



Are the triangles similar? If so, write a similarity statement and name the postulate or theorem you used. If not, explain.

11.


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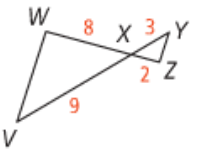

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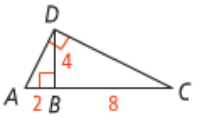

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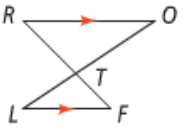
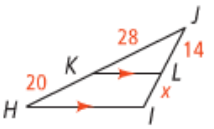


Algebra **Explain why the triangles are similar. Then find the value of x.**

15.


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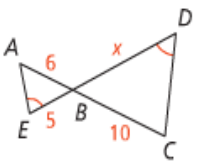


17. In a garden, a birdbath 2 ft 6 in. tall casts an 18-in. shadow at the same time an oak tree casts a 90-ft shadow. How tall is the oak tree?

Do you UNDERSTAND?

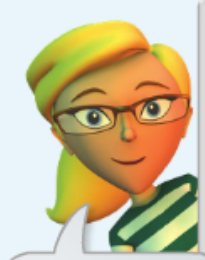
18. **Writing** You find an old scale drawing of your home, but the scale has faded and you cannot read it. How can you find the scale of the drawing?

19. **Reasoning** The sides of one triangle are twice as long as the corresponding sides of a second triangle. What is the relationship between the angles?

20. **Error Analysis** Your classmate says that since all congruent polygons are similar, all similar polygons must be congruent. Is he right? Explain.

7-4 Similarity in Right Triangles

Objective To find and use relationships in similar right triangles

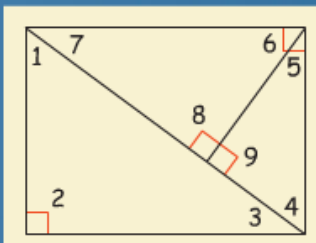


This activity will help you visualize the theorem and corollaries in this lesson.

SOLVE IT!

Getting Ready!

Draw a diagonal of a rectangular piece of paper to form two right triangles. In one triangle, draw the altitude from the right angle to the hypotenuse. Number the angles as shown. Cut out the three triangles. How can you match the angles of the triangles to show that all three triangles are similar? Explain how you know the matching angles are congruent.



Dynamic Activity
Similarity in Right Triangles

In the Solve It, you looked at three similar right triangles. In this lesson, you will learn new ways to think about the proportions that come from these similar triangles. You began with three separate, nonoverlapping triangles in the Solve It. Now you will see the two smaller right triangles fitting side-by-side to form the largest right triangle.

Lesson Vocabulary
- geometric mean

Essential Understanding When you draw the *altitude to the hypotenuse* of a right triangle, you form three pairs of similar right triangles.

take note

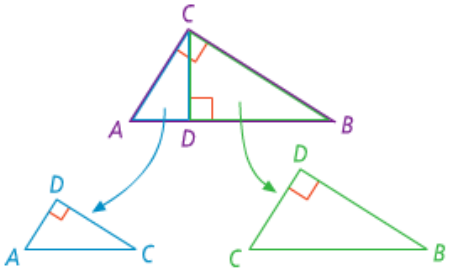
Theorem 7-3

Theorem

The altitude to the hypotenuse of a right triangle divides the triangle into two triangles that are similar to the original triangle and to each other.

If . . .

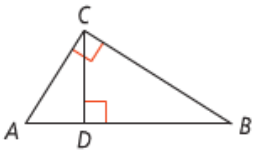
$\triangle ABC$ is a right triangle with right $\angle ACB$, and $\overline{CD}$ is the altitude to the hypotenuse



Then . . .

$\triangle ABC \sim \triangle ACD$
$\triangle ABC \sim \triangle CBD$
$\triangle ACD \sim \triangle CBD$

Proof **Proof of Theorem 7-3**



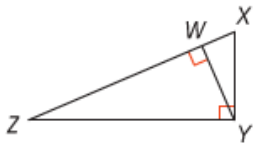
Given: Right $\triangle ABC$ with right $\angle ACB$ and altitude $\overline{CD}$

Prove: $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$, $\triangle ACD \sim \triangle CBD$

Statements	Reasons
1) $\angle ACB$ is a right angle.	1) Given
2) $\overline{CD}$ is an altitude.	2) Given
3) $\overline{CD} \perp \overline{AB}$	3) Definition of altitude
4) $\angle ADC$ and $\angle CDB$ are right angles.	4) Definition of $\perp$
5) $\angle ADC \cong \angle ACB$, $\angle CDB \cong \angle ACB$	5) All right $\triangle$ are $\cong$.
6) $\angle A \cong \angle A$, $\angle B \cong \angle B$	6) Reflexive Property of $\cong$
7) $\triangle ACD \sim \triangle ABC$, $\triangle CBD \sim \triangle ABC$	7) AA $\sim$ Postulate
8) $\angle ACD \cong \angle B$	8) Corresponding $\triangle$ of $\sim$ $\triangle$ are $\cong$.
9) $\angle ADC \cong \angle CDB$	9) All right $\triangle$ are $\cong$.
10) $\triangle ACD \sim \triangle CBD$	10) AA $\sim$ Postulate

Problem 1 Identifying Similar Triangles

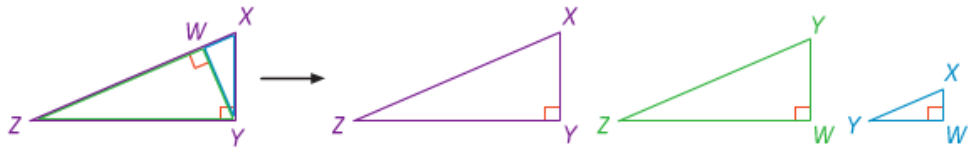
What similarity statement can you write relating the three triangles in the diagram?



Plan

What will help you see the corresponding vertices?
Sketch the triangles separately in the same orientation.

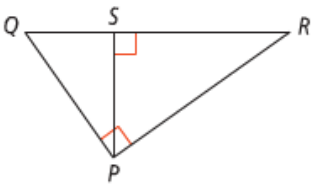
$\overline{YW}$ is the altitude to the hypotenuse of right $\triangle XYZ$, so you can use Theorem 7-3. There are three similar triangles.



$\triangle XYZ \sim \triangle YWZ \sim \triangle XWY$

Got It? **1. a.** What similarity statement can you write relating the three triangles in the diagram?

b. Reasoning From the similarity statement in part (a), write two different proportions using the ratio $\frac{SR}{SP}$.



Proportions in which the means are equal occur frequently in geometry. For any two positive numbers a and b, the **geometric mean** of a and b is the positive number x such that $\frac{a}{x} = \frac{x}{b}$.

Problem 2 Finding the Geometric Mean

Multiple Choice What is the geometric mean of 6 and 15?

(A) 90 (B) $3\sqrt{10}$ (C) $9\sqrt{10}$ (D) 30

Think

How do you use the definition of geometric mean?
Set up a proportion with x in both means positions. The numbers 6 and 15 go into the extremes positions.

$\frac{6}{x} = \frac{x}{15}$ Definition of geometric mean

$x^2 = 90$ Cross Products Property

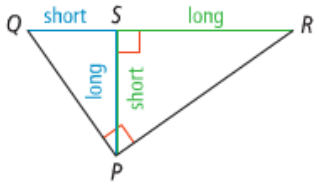
$x = \sqrt{90}$ Take the positive square root of each side.

$x = 3\sqrt{10}$ Write in simplest radical form.

The geometric mean of 6 and 15 is $3\sqrt{10}$. The correct answer is B.

Got It? **2.** What is the geometric mean of 4 and 18?

In Got It 1 part (b), you used a pair of similar triangles to write a proportion with a geometric mean.



$\triangle SQP \sim \triangle SPR$

$\frac{\text{short leg}}{\text{short leg}} = \frac{\text{long leg}}{\text{long leg}}$

$\frac{SQ}{SP} = \frac{SP}{SR}$

SP is the geometric mean of *SQ* and *SR*.

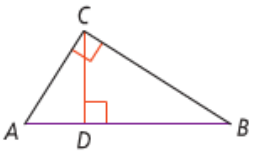
This illustrates the first of two important corollaries of Theorem 7-3.

take note

Corollary 1 to Theorem 7-3

Corollary
The length of the altitude to the hypotenuse of a right triangle is the geometric mean of the lengths of the segments of the hypotenuse.

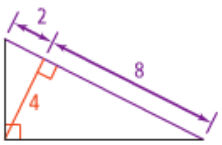
If . . .



Then . . .

$\frac{AD}{CD} = \frac{CD}{DB}$

Example



Segments of hypotenuse → $\frac{2}{4} = \frac{4}{8}$ ← Altitude to hypotenuse

You will prove Corollary 1 in Exercise 42.

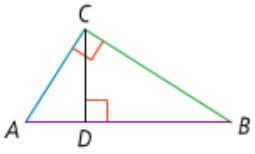
Take note

Corollary 2 to Theorem 7-3

Corollary

The altitude to the hypotenuse of a right triangle separates the hypotenuse so that the length of each leg of the triangle is the geometric mean of the length of the hypotenuse and the length of the segment of the hypotenuse adjacent to the leg.

If . . .

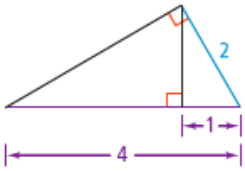


Then . . .

$\frac{AB}{AC} = \frac{AC}{AD}$

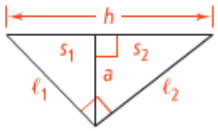
$\frac{AB}{CB} = \frac{CB}{DB}$

Example



You will prove Corollary 2 in Exercise 43.

The corollaries to Theorem 7-3 give you ways to write proportions using lengths in right triangles without thinking through the similar triangles. To help remember these corollaries, consider the diagram and these properties.



Corollary 1

$\frac{s_1}{a} = \frac{a}{s_2}$

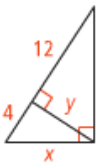
Corollary 2

$\frac{h}{\ell_1} = \frac{\ell_1}{s_1}, \frac{h}{\ell_2} = \frac{\ell_2}{s_2}$

ONLINE PROBLEM

Problem 3 Using the Corollaries

Algebra What are the values of x and y?

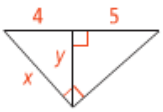


Plan

How do you decide which corollary to use?
If you are using or finding an altitude, use Corollary 1. If you are using or finding a leg or hypotenuse, use Corollary 2.

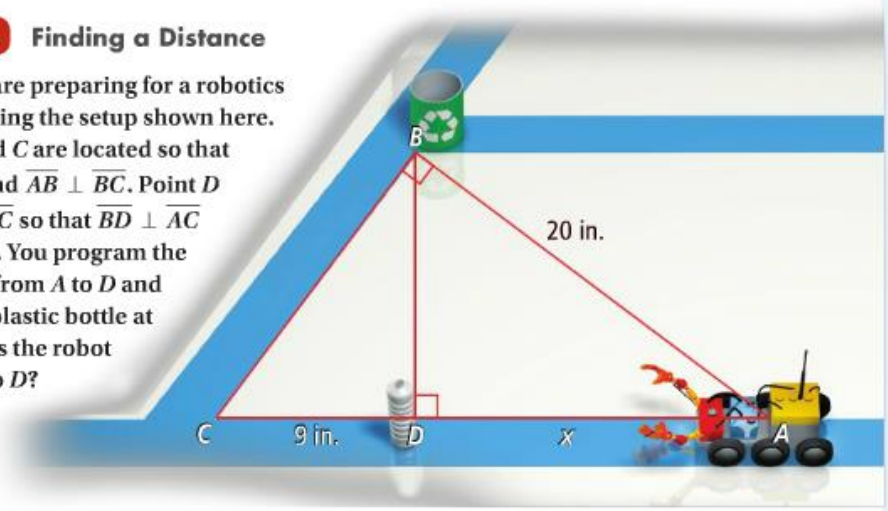
Use Corollary 2.			Use Corollary 1.
$\frac{4}{x} = \frac{x}{4+12}$	Write a proportion.	$\frac{4}{y} = \frac{y}{12}$	
$x^2 = 64$	Cross Products Property	$y^2 = 48$	
$x = \sqrt{64}$	Take the positive square root.	$y = \sqrt{48}$	
$x = 8$	Simplify.	$y = 4\sqrt{3}$	

Got It? **3.** What are the values of x and y?



Problem 4 Finding a Distance

Robotics You are preparing for a robotics competition using the setup shown here. Points A, B, and C are located so that $AB = 20$ in., and $\overline{AB} \perp \overline{BC}$. Point D is located on $\overline{AC}$ so that $\overline{BD} \perp \overline{AC}$ and $DC = 9$ in. You program the robot to move from A to D and to pick up the plastic bottle at D. How far does the robot travel from A to D?



Think

You can't solve this equation by taking the square root. What do you do?
Write the quadratic equation in the standard form $ax^2 + bx + c = 0$. Then solve by factoring or use the quadratic formula.

$\frac{x+9}{20} = \frac{20}{x}$			Corollary 2
$x^2 + 9x = 400$			Cross Products Property
$x^2 + 9x - 400 = 0$			Subtract 400 from each side.
$(x - 16)(x + 25) = 0$			Factor.
$x - 16 = 0$	or	$(x + 25) = 0$	Zero-Product Property
$x = 16$	or	$x = -25$	Solve for x.

Only the positive solution makes sense in this situation. The robot travels 16 in.

Got It? **4.** From point D, the robot must turn right and move to point B to put the bottle in the recycling bin. How far does the robot travel from D to B?

Lesson Check

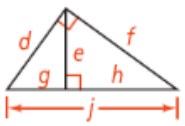
Do you know HOW?

Find the geometric mean of each pair of numbers.

1. 4 and 9

2. 4 and 12

Use the figure to complete each proportion.



3. $\frac{g}{e} = \frac{e}{\square}$

4. $\frac{j}{d} = \frac{d}{\square}$

5. $\frac{\square}{f} = \frac{f}{\square}$

6. $\frac{j}{\square} = \frac{\square}{g}$

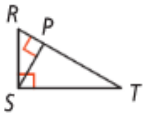
Do you UNDERSTAND?

7. Vocabulary Identify the following in $\triangle RST$.

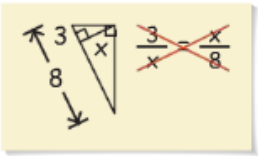
a. the hypotenuse

b. the segments of the hypotenuse

c. the segment of the hypotenuse adjacent to leg $\overline{ST}$



8. Error Analysis A classmate wrote an incorrect proportion to find x. Explain and correct the error.

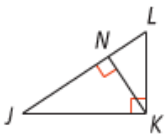


Practice and Problem-Solving Exercises

A Practice

Write a similarity statement relating the three triangles in each diagram. See Problem 1.

9.



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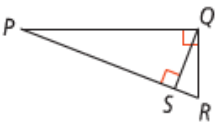

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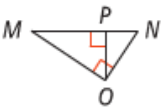


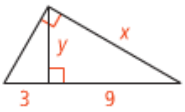
Algebra Find the geometric mean of each pair of numbers. See Problem 2.

12. 4 and 10 **13.** 3 and 48 **14.** 5 and 125

15. 7 and 9 **16.** 3 and 16 **17.** 4 and 49

Algebra Solve for x and y. See Problems 3 and 4.

18.



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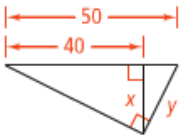

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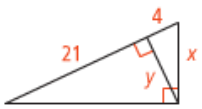
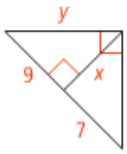
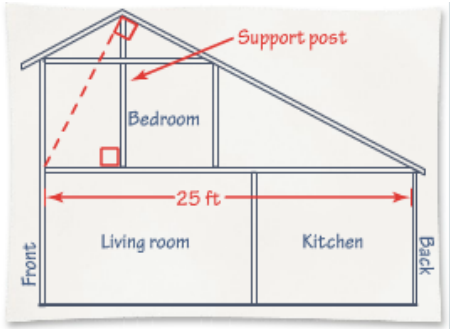


21.



22. Architecture The architect's side view drawing of a saltbox-style house shows a post that supports the roof ridge. The support post is 10 ft tall. How far from the front of the house is the support post positioned?



B Apply

23. **a.** The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments 2 cm and 8 cm long. Find the length of the altitude to the hypotenuse.

b. Use a ruler to make an accurate drawing of the right triangle in part (a).

c. Writing Describe how you drew the triangle in part (b).

Algebra Find the geometric mean of each pair of numbers.

24. 1 and 1000 **25.** 5 and 1.25 **26.** $\sqrt{8}$ and $\sqrt{2}$ **27.** $\frac{1}{2}$ and 2 **28.** $\sqrt{28}$ and $\sqrt{7}$

29. Reasoning A classmate says the following statement is true: The geometric mean of positive numbers a and b is $\sqrt{ab}$. Do you agree? Explain.

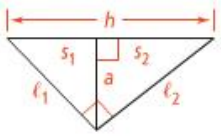
30. Think About a Plan The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments with lengths in the ratio 1 : 2. The length of the altitude is 8. How long is the hypotenuse?

- How can you use the given ratio to help you draw a sketch of the triangle?
- How can you use the given ratio to write expressions for the lengths of the segments of the hypotenuse?
- Which corollary to Theorem 7-3 applies to this situation?

31. Archaeology To estimate the height of a stone figure, Anya holds a small square up to her eyes and walks backward from the figure. She stops when the bottom of the figure aligns with the bottom edge of the square and the top of the figure aligns with the top edge of the square. Her eye level is 1.84 m from the ground. She is 3.50 m from the figure. What is the height of the figure to the nearest hundredth of a meter?

32. Reasoning Suppose the altitude to the hypotenuse of a right triangle bisects the hypotenuse. How does the length of the altitude compare with the lengths of the segments of the hypotenuse? Explain.

The diagram shows the parts of a right triangle with an altitude to the hypotenuse. For the two given measures, find the other four. Use simplest radical form.

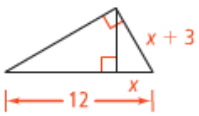


33. $h = 2, s_1 = 1$ **34.** $a = 6, s_1 = 6$ **35.** $\ell_1 = 2, s_2 = 3$ **36.** $s_1 = 3, \ell_2 = 6\sqrt{3}$

37. Coordinate Geometry $\overline{CD}$ is the altitude to the hypotenuse of right $\triangle ABC$. The coordinates of A, D, and B are (4, 2), (4, 6), and (4, 15), respectively. Find all possible coordinates of point C.

Algebra **Find the value of x.**

38.



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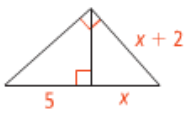

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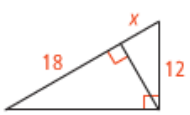

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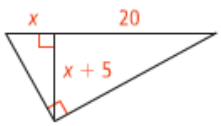
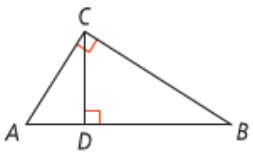


Use the figure at the right for Exercises 42 and 43.



42. Proof Prove Corollary 1 to Theorem 7-3.

Given: Right $\triangle ABC$ with altitude to the hypotenuse $\overline{CD}$

Prove: $\frac{AD}{CD} = \frac{CD}{DB}$

43. Proof Prove Corollary 2 to Theorem 7-3.

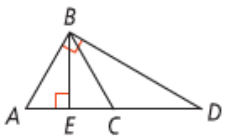
Given: Right $\triangle ABC$ with altitude to the hypotenuse $\overline{CD}$

Prove: $\frac{AB}{AC} = \frac{AC}{AD}, \frac{AB}{BC} = \frac{BC}{DB}$

Challenge

44. Proof **Given:** Right $\triangle ABD$ with altitude to the hypotenuse $\overline{BE}$, and equilateral $\triangle ABC$

Prove: $BE = AE\sqrt{3}$



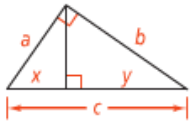
45. a. Consider the following conjecture: The product of the lengths of the two legs of a right triangle is equal to the product of the lengths of the hypotenuse and the altitude to the hypotenuse. Draw a figure for the conjecture. Write the *Given* information and what you are to *Prove*.

b. Reasoning Is the conjecture true? Explain.

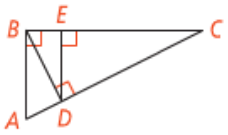
46. **a.** In the diagram, $c = x + y$. Use Corollary 2 to Theorem 7-3 to write two more equations involving a, b, c, x, and y.

b. The equations in part (a) form a system of three equations in five variables. Reduce the system to one equation in three variables by eliminating x and y.

c. State in words what the one resulting equation tells you.



47. Proof **Given:** In right $\triangle ABC$, $\overline{BD} \perp \overline{AC}$, and $\overline{DE} \perp \overline{BC}$.
Prove: $\frac{AD}{DC} = \frac{BE}{EC}$



Standardized Test Prep

SAT/ACT

48. The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of lengths 5 and 15. What is the length of the altitude?

(A) 3 (B) $5\sqrt{3}$ (C) 10 (D) $5\sqrt{5}$

49. A triangle has side lengths 3 in., 4 in., and 6 in. The longest side of a similar triangle is 15 in. What is the length of the shortest side of the similar triangle?

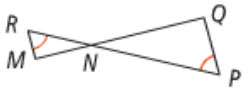
(F) 1 in. (G) 1.2 in. (H) 7.5 in. (I) 10 in.

Short Response

50. Two students disagree about the measures of angles in a kite. They know that two angles measure 124 and 38. But they get different answers for the other two angles. Can they both be correct? Explain.

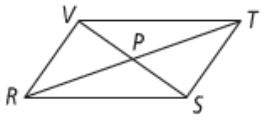
Mixed Review

51. Write a similarity statement for the two triangles. How do you know they are similar?



See Lesson 7-3.

Algebra **Find the values of x and y in $\square RSTV$.**



See Lesson 6-2.

52. $RP = 2x$, $PT = y + 2$, $VP = y$, $PS = x + 3$

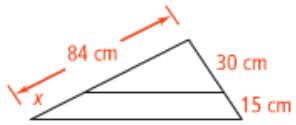
53. $RV = 2x + 3$, $VT = 5x$, $TS = y + 5$, $SR = 4y - 1$

Get Ready! **To prepare for Lesson 7-5, do Exercises 54–56.**

The two triangles in each diagram are similar. Find the value of x in each.

See Lesson 7-2.

54.



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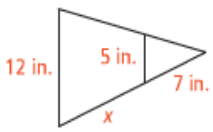

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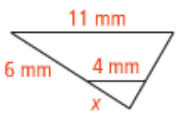


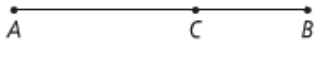
Concept Byte

Use With Lesson 7-4

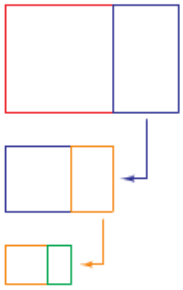
ACTIVITY

The Golden Ratio

In his book *Elements*, Euclid defined the *extreme and mean ratio* using a proportion formed by dividing a line segment at a particular point, as shown at the right. In the diagram, C divides $\overline{AB}$ so that the length of $\overline{AC}$ is the geometric mean of the lengths of $\overline{AB}$ and $\overline{CB}$. That is, $\frac{AB}{AC} = \frac{AC}{CB}$. The ratio $\frac{AC}{CB}$ is known today as the *golden ratio*, which is about 1.618 : 1.



Rectangles in which the ratio of the length to the width is the golden ratio are *golden rectangles*. A golden rectangle can be divided into a square and a rectangle that is similar to the original rectangle. A pattern of golden rectangles is shown at the right.



Activity 1

To derive the golden ratio, consider $\overline{AB}$ divided by C so that $\frac{AB}{AC} = \frac{AC}{CB}$.

1. Use the diagram at the right to write a proportion that relates the lengths of the segments. How can you rewrite the proportion as a quadratic equation?
2. Use the quadratic formula to solve the quadratic equation in Question 1. Why does only one solution makes sense in this situation?
3. What is the value of x to the nearest ten-thousandth? Use a calculator.

Spiral growth patterns of sunflower seeds and the spacing of plant leaves on the stem are two examples of the golden ratio and the Fibonacci sequence in nature.

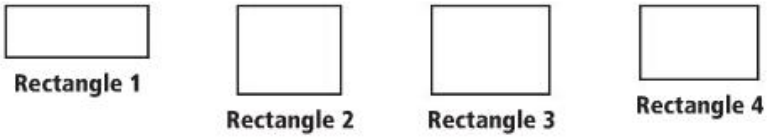
Activity 2

In the Fibonacci sequence, each term after the first two terms is the sum of the preceding two terms. The first six terms of the Fibonacci sequence are 1, 1, 2, 3, 5, and 8.

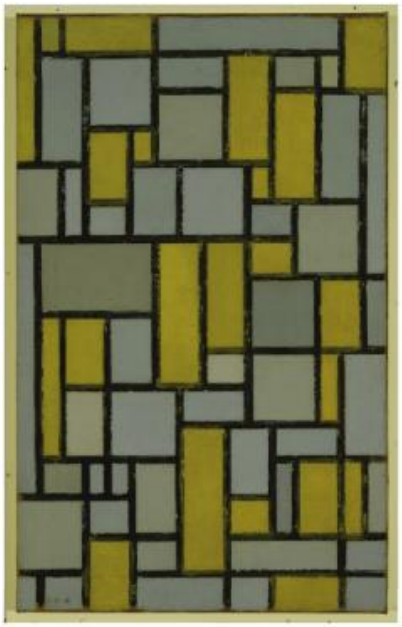
4. What are the next nine terms of the Fibonacci sequence?
5. Starting with the second term, the ratios of each term to the previous term for the first six terms are $\frac{1}{1} = 1$, $\frac{2}{1} = 2$, $\frac{3}{2} = 1.5$, $\frac{5}{3} = 1.666\ldots$, and $\frac{8}{5} = 1.6$. What are the next nine ratios rounded to the nearest thousandth?
6. Compare the ratios you found in Question 5. What do you notice? How is the Fibonacci sequence related to the golden ratio?

Exercises

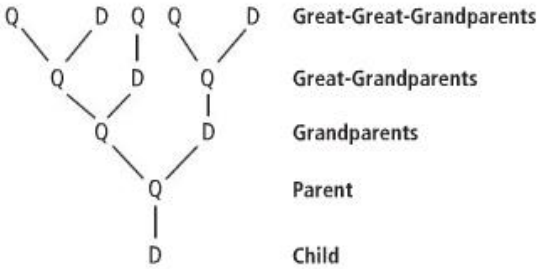
7. The golden rectangle is considered to be pleasing to the human eye. Of the following rectangles, which do you prefer? Is it a golden rectangle?



8. Some believe that Piet Mondrian used golden rectangles in his paintings. The painting at the right is called *Composition with Gray and Light Brown*. Use a centimeter ruler to measure the length and width of the picture of the painting. Is it approximately a golden rectangle? Explain.



9. A drone is a male honeybee. Drones have only one parent, a queen. Workers and queens are female honeybees. Females have two parents, a drone and a queen. Part of the family tree showing the ancestors of a drone is shown below, where D represents a drone and Q represents a queen.

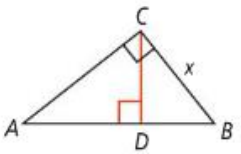


a. Continue the family tree for three more generations of ancestors.

b. Count the number of honeybees in each generation. What pattern do you notice?

10. What is the relationship between the flowers and the Fibonacci sequence?

11. In $\triangle ABC$, point D divides the hypotenuse into the golden ratio. That is, $AD : DB$ is about $1.618 : 1$. $\overline{CD}$ is an altitude. Using the value 1.618 for AD and the value 1 for DB, solve for x. What do you notice?



Concept Byte

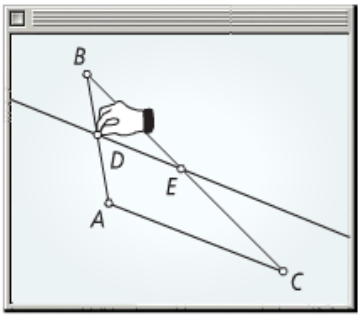
Use With Lesson 7-5

TECHNOLOGY

Exploring Proportions in Triangles

Activity 1

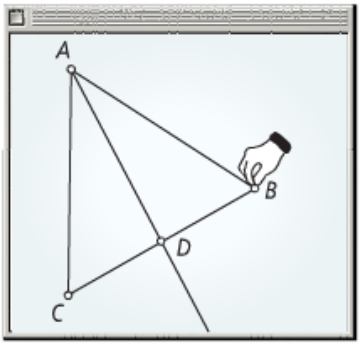
Use geometry software to draw $\triangle ABC$. Construct point D on $\overline{AB}$. Next, construct a line through D parallel to $\overline{AC}$. Then construct the intersection E of the parallel line with $\overline{BC}$.



1. Measure $\overline{BD}$, $\overline{DA}$, $\overline{BE}$, and $\overline{EC}$. Calculate the ratios $\frac{BD}{DA}$ and $\frac{BE}{EC}$.
2. Manipulate $\triangle ABC$ and observe $\frac{BD}{DA}$ and $\frac{BE}{EC}$. What do you notice?
3. Make a conjecture about the four segments formed by a line parallel to one side of a triangle intersecting the other two sides.

Activity 2

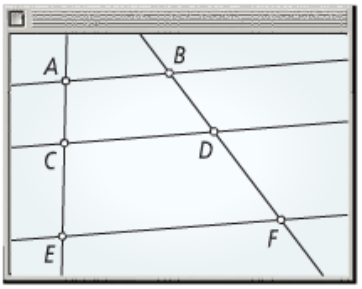
Use geometry software to draw $\triangle ABC$. Construct the bisector of $\angle A$. Label the intersection of the bisector and $\overline{CB}$ with the letter D.



4. Measure $\overline{AC}$, $\overline{AB}$, $\overline{CD}$, and $\overline{BD}$. Calculate the ratios $\frac{AC}{AB}$ and $\frac{CD}{BD}$.
5. Manipulate $\triangle ABC$ and observe $\frac{AC}{AB}$ and $\frac{CD}{BD}$. What do you notice?
6. Make a conjecture about the two sides of a triangle and the two segments formed by the bisector of the opposite angle.

Exercises

7. Construct $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD} \parallel \overleftrightarrow{EF}$. Then construct two transversals that intersect all three parallel lines. Measure $\overline{AC}$, $\overline{CE}$, $\overline{BD}$, and $\overline{DF}$. Calculate the ratios $\frac{AC}{CE}$ and $\frac{BD}{DF}$. Manipulate the locations of A and B and observe $\frac{AC}{CE}$ and $\frac{BD}{DF}$. Make a conjecture about the segments of the transversals formed by the three parallel lines intersecting two transversals.
8. Suppose four or more parallel lines intersect two transversals. Make a conjecture about the segments of the transversals.



7-5 Proportions in Triangles

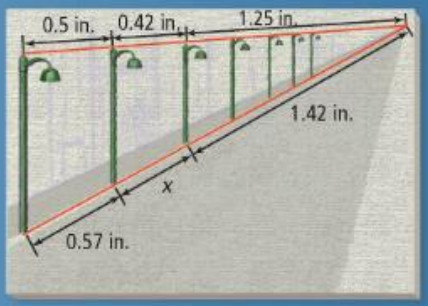
Objective To use the Side-Splitter Theorem and the Triangle-Angle-Bisector Theorem

Remember to keep things in perspective and in proportion!

SOLVE IT!

Getting Ready!

An artist uses perspective to draw parallel lampposts along a city street, as shown in the diagram. What is the value of x? Justify your answer.



The Solve It involves parallel lines cut by two transversals that intersect. In this lesson, you will learn how to use proportions to find lengths of segments formed by parallel lines that intersect two or more transversals.

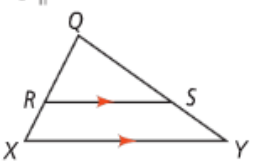
Essential Understanding When two or more parallel lines intersect other lines, proportional segments are formed.

take note

Theorem 7-4 Side-Splitter Theorem

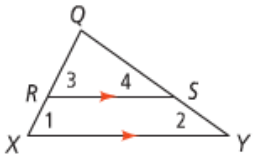
Theorem
If a line is parallel to one side of a triangle and intersects the other two sides, then it divides those sides proportionally.

If . . .
$\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$



Then . . .
$\frac{XR}{RQ} = \frac{YS}{SQ}$

Proof **Proof of Theorem 7-4: Side-Splitter Theorem**



Given: $\triangle QXY$ with $\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$

Prove: $\frac{XR}{RQ} = \frac{YS}{SQ}$

Statements	Reasons
1) $\overleftrightarrow{RS} \parallel \overleftrightarrow{XY}$	1) Given
2) $\angle 1 \cong \angle 3, \angle 2 \cong \angle 4$	2) If lines are $\parallel$, then corresponding $\measuredangle$s are $\cong$.
3) $\triangle QXY \sim \triangle QRS$	3) AA $\sim$ Postulate
4) $\frac{XQ}{RQ} = \frac{YQ}{SQ}$	4) Corresponding sides of $\sim$ $\triangle$s are proportional.
5) $XQ = XR + RQ$, $YQ = YS + SQ$	5) Segment Addition Postulate
6) $\frac{XR + RQ}{RQ} = \frac{YS + SQ}{SQ}$	6) Substitution Property
7) $\frac{XR}{RQ} = \frac{YS}{SQ}$	7) Property of Proportions (3)

Problem 1 Using the Side-Splitter Theorem

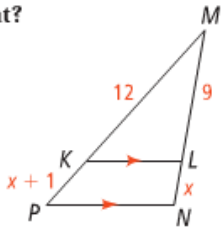
GRIDDED RESPONSE

What is the value of x in the diagram at the right?

Plan

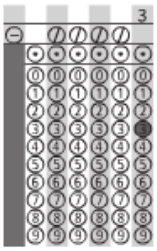
How can you use the parallel lines in the diagram?

$\overline{KL}$ is parallel to one side of $\triangle MNP$. Use the Side-Splitter Theorem to set up a proportion.

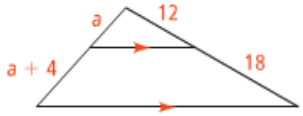


$\frac{PK}{KM} = \frac{NL}{LM}$		Side-Splitter Theorem
$\frac{x+1}{12} = \frac{x}{9}$		Substitute.
$9x + 9 = 12x$		Cross Products Property
$9 = 3x$		Subtract $9x$ from each side.
$3 = x$		Divide each side by 3.

Grid in the number 3.



Got It? **1. a.** What is the value of a in the diagram at the right?



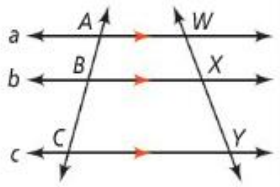
b. Reasoning In $\triangle XYZ$, $\overline{RS}$ joins $\overline{XY}$ and $\overline{YZ}$ with R on $\overline{XY}$ and S on $\overline{YZ}$, and $\overline{RS} \parallel \overline{XZ}$. If $\frac{YR}{RX} = \frac{YS}{SZ} = 1$, what must be true about RS? Justify your reasoning.

take note

Corollary Corollary to the Side-Splitter Theorem

Corollary
If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

If . . .
$a \parallel b \parallel c$



Then . . .
$\frac{AB}{BC} = \frac{WX}{XY}$

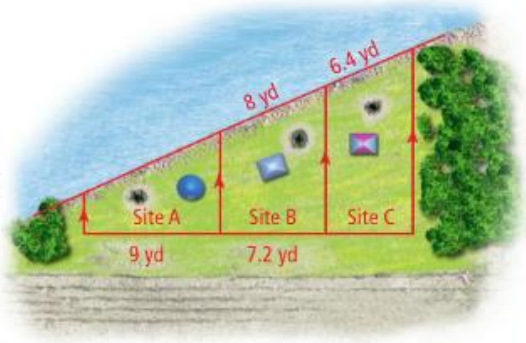
You will prove the Corollary to Theorem 7-4 in Exercise 46.

Plan

What information does the diagram give you?
The lines separating the campsites are parallel. Think of the river and the edge of the road as transversals. Then the boundaries along the road and river for each campsite are proportional.

Problem 2 Finding a Length

Camping **Three campsites are shown in the diagram. What is the length of Site A along the river?**



Let x be the length of Site A along the river.

$\frac{x}{8} = \frac{9}{7.2}$ Corollary to the Side-Splitter Theorem

$7.2x = 72$ Cross Products Property

$x = 10$ Divide each side by 7.2.

The length of Site A along the river is 10 yd.

Got It? **2.** What is the length of Site C along the road?

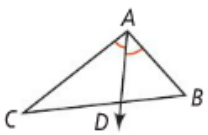
Essential Understanding The bisector of an angle of a triangle divides the opposite side into two segments with lengths proportional to the sides of the triangle that form the angle.

take note

Theorem 7-5 Triangle-Angle-Bisector Theorem

Theorem
If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

If . . .
$\overrightarrow{AD}$ bisects $\angle CAB$

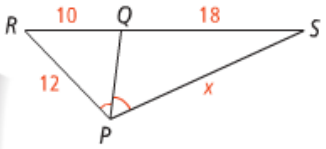


Then . . .
$\frac{CD}{DB} = \frac{CA}{BA}$

You will prove the Triangle-Angle-Bisector Theorem in Exercise 47.

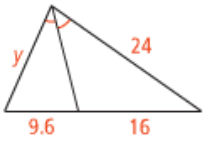
Problem 3 Using the Triangle-Angle-Bisector Theorem

Algebra What is the value of x in the diagram at the right?



Think	Write
$\overline{PQ}$ bisects $\angle RPS$. Use the Triangle-Angle-Bisector Theorem to write a proportion.	$\frac{RQ}{QS} = \frac{PR}{PS}$
Substitute corresponding side lengths in the proportion.	$\frac{10}{18} = \frac{12}{x}$
Use the Cross Products Property.	$10x = 216$
Divide each side by 10.	$x = 21.6$

Got It? **3.** What is the value of y in the diagram at the right?

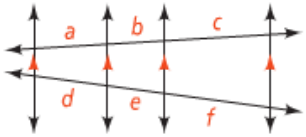


Lesson Check

Do you know HOW?

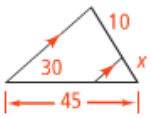
Use the figure to complete each proportion.

1. $\frac{a}{b} = \frac{\blacksquare}{e}$ 2. $\frac{b}{\blacksquare} = \frac{e}{f}$
3. $\frac{a}{b+c} = \frac{\blacksquare}{e+f}$



What is the value of x in each figure?

4.



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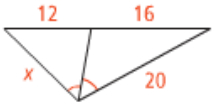
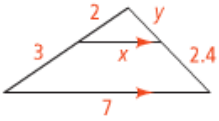


Do you UNDERSTAND?

6. **Compare and Contrast** How is the Corollary to the Side-Splitter Theorem related to Theorem 6-7: If three (or more) parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal?

7. **Compare and Contrast** How are the Triangle-Angle-Bisector Theorem and Corollary 1 to Theorem 7-3 alike? How are they different?

8. **Error Analysis** A classmate says you can use the Side-Splitter Theorem to find both x and y in the diagram. Explain what is wrong with your classmate's statement.

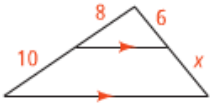


Practice and Problem-Solving Exercises

A Practice

Algebra Solve for x. See Problem 1.

9.



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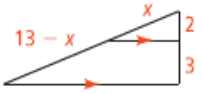

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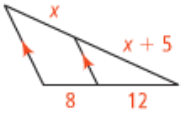

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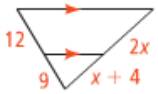
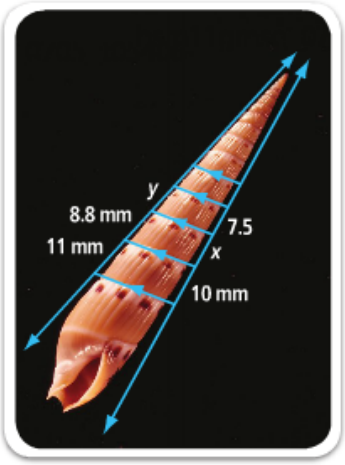


Marine Biology Use the information shown on the auger shell. See Problem 2.

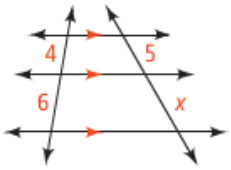
13. What is the value of x?

14. What is the value of y?



Algebra Solve for x.

15.



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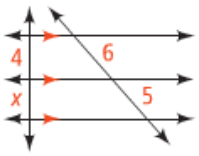

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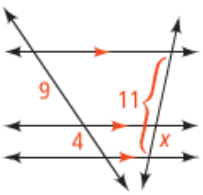

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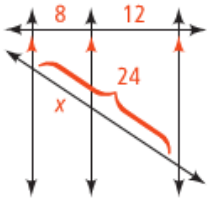
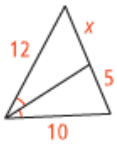


Algebra Solve for x. See Problem 3.

19.



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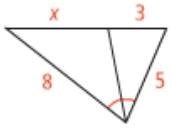

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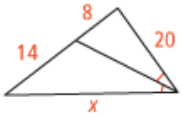

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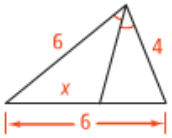
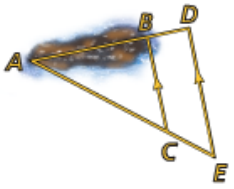


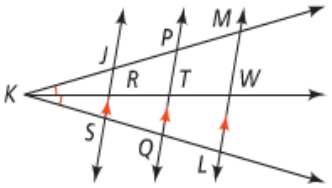
23. Writing The size of an oil spill on the open ocean is difficult to measure directly. Use the figure at the right to describe how you could find the length of the oil spill indirectly. What measurements and calculations would you use?



24. The lengths of the sides of a triangle are 5 cm, 12 cm, and 13 cm. Find the lengths, to the nearest tenth, of the segments into which the bisector of each angle divides the opposite side.

B Apply

Use the figure at the right to complete each proportion. Justify your answer.



25. $\frac{RS}{\blacksquare} = \frac{JR}{KJ}$

26. $\frac{KJ}{JP} = \frac{KS}{\blacksquare}$

27. $\frac{QL}{PM} = \frac{SQ}{\blacksquare}$

28. $\frac{PT}{\blacksquare} = \frac{TQ}{KQ}$

29. $\frac{KL}{LW} = \frac{\blacksquare}{MW}$

30. $\frac{\blacksquare}{KP} = \frac{LQ}{KQ}$

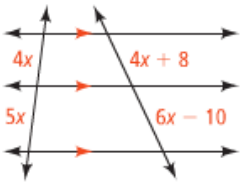
Urban Design **In Washington, D.C., E. Capitol Street, Independence Avenue, C Street, and D Street are parallel streets that intersect Kentucky Avenue and 12th Street.**

31. How long (to the nearest foot) is Kentucky Avenue between C Street and D Street?

32. How long (to the nearest foot) is Kentucky Avenue between E. Capitol Street and Independence Avenue?

Algebra **Solve for x.**

33.



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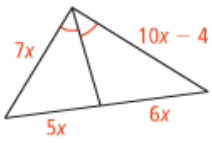

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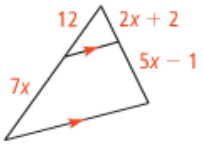
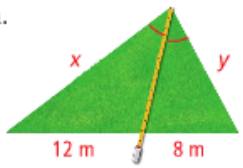


36. **Think About a Plan** The perimeter of the triangular lot at the right is 50 m. The surveyor's tape bisects an angle. Find the lengths x and y.

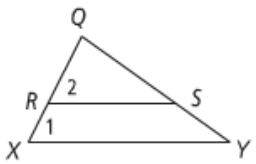
- How can you use the perimeter to write an equation in x and y?
- What other relationship do you know between x and y?



37. Proof Prove the Converse of the Side-Splitter Theorem: If a line divides two sides of a triangle proportionally, then it is parallel to the third side.

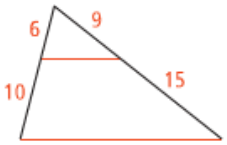
Given: $\frac{XR}{RQ} = \frac{YS}{SQ}$

Prove: $\overline{RS} \parallel \overline{XY}$



Determine whether the red segments are parallel. Explain each answer. You can use the theorem proved in Exercise 37.

38.



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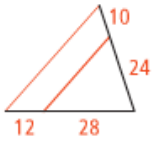

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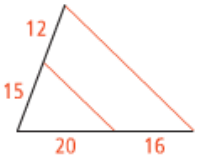


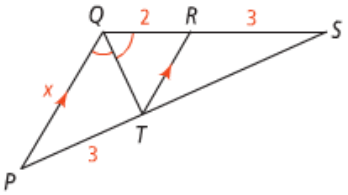
41. An angle bisector of a triangle divides the opposite side of the triangle into segments 5 cm and 3 cm long. A second side of the triangle is 7.5 cm long. Find all possible lengths for the third side of the triangle.

42. Open-Ended In a triangle, the bisector of an angle divides the opposite side into two segments with lengths 6 cm and 9 cm. How long could the other two sides of the triangle be? (*Hint:* Make sure the three sides satisfy the Triangle Inequality Theorem.)

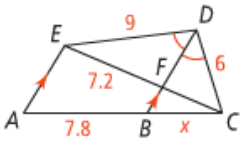
43. Reasoning In $\triangle ABC$, the bisector of $\angle C$ bisects the opposite side. What type of triangle is $\triangle ABC$? Explain your reasoning.

Algebra **Solve for x.**

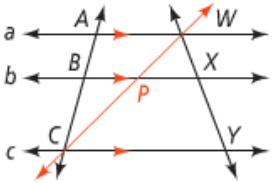
44.



45.



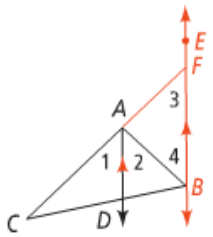
46. Proof Prove the Corollary to the Side-Splitter Theorem. In the diagram from page 473, draw the auxiliary line $\overleftrightarrow{CW}$ and label its intersection with line b as point P.



Given: $a \parallel b \parallel c$

Prove: $\frac{AB}{BC} = \frac{WX}{XY}$

47. Proof Prove the Triangle-Angle-Bisector Theorem. In the diagram from page 473, draw the auxiliary line $\overleftrightarrow{BE}$ so that $\overleftrightarrow{BE} \parallel \overline{DA}$. Extend $\overline{CA}$ to meet $\overleftrightarrow{BE}$ at point F.



Given: $\overleftrightarrow{AD}$ bisects $\angle CAB$.

Prove: $\frac{CD}{DB} = \frac{CA}{BA}$

Challenge

48. Use the definition in part (a) to prove the statements in parts (b) and (c).

a. Write a definition for a midsegment of a parallelogram.

b. A parallelogram midsegment is parallel to two sides of the parallelogram.

c. A parallelogram midsegment bisects the diagonals of a parallelogram.

49. State the converse of the Triangle-Angle-Bisector Theorem. Give a convincing argument that the converse is true or a counterexample to prove that it is false.

50. In $\triangle ABC$, the bisectors of $\angle A$, $\angle B$, and $\angle C$ cut the opposite sides into lengths a_1 and a_2, b_1 and b_2, and c_1 and c_2, respectively, labeled in order counterclockwise around $\triangle ABC$. Find the perimeter of $\triangle ABC$ for each set of values.

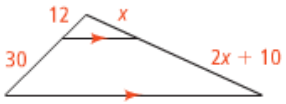
a. $b_1 = 16, b_2 = 20, c_1 = 18$ **b.** $a_1 = \frac{5}{3}, a_2 = \frac{10}{3}, b_1 = \frac{15}{4}$

Standardized Test Prep

GRIDDED RESPONSE

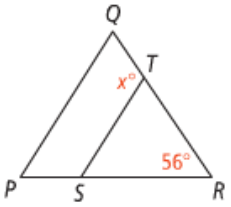
SAT/ACT

51. What is the value of x in the figure at the right?



52. Suppose $\triangle VLQ \sim \triangle PSX$. If $m\angle V = 48$ and $m\angle L = 80$, what is $m\angle X$?

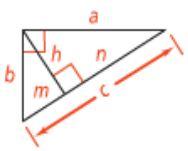
53. In the diagram at the right, $\overline{PR} \cong \overline{QR}$. For what value of x is $\overline{TS}$ parallel to $\overline{QP}$?



54. Leah is playing basketball on an outdoor basketball court. The 10-ft pole supporting the basketball net casts a 15-ft shadow. At the same time, the length of Leah's shadow is 8 ft 3 in. What is Leah's height in inches? You can assume both Leah and the pole supporting the net are perpendicular to the ground.

Mixed Review

Use the figure to complete each proportion. See Lesson 7-4.



55. $\frac{n}{h} = \frac{h}{\blacksquare}$ **56.** $\frac{\blacksquare}{b} = \frac{b}{c}$

57. $\frac{n}{a} = \frac{a}{\blacksquare}$ **58.** $\frac{m}{h} = \frac{\blacksquare}{n}$

Find the center of the circle that you can circumscribe about each $\triangle ABC$. See Lesson 5-3.

59. $A(0, 0)$, $B(6, 0)$, $C(0, -6)$

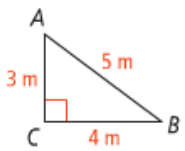
60. $A(2, 5)$, $B(-2, 5)$, $C(-2, -1)$

61. $A(-2, 0)$, $B(5, 5)$, $C(-2, 5)$

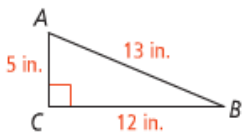
Get Ready! **To prepare for Lesson 8-1, do Exercises 62–64.**

Square the lengths of the sides of each triangle. See p. 829.

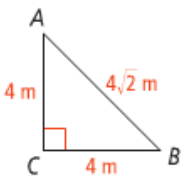
62.



63.

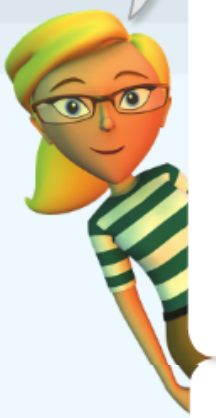


64.



7 Pull It All Together

To solve these problems, you will pull together many concepts and skills that you have studied about similarity.

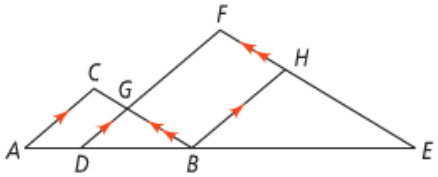


BIG idea Visualization, Reasoning and Proof, and Similarity

You can show that two triangles are similar when certain relationships exist between two or three pairs of corresponding parts. If you know two triangles are similar, then you know their corresponding sides are proportional.

Task 1

In the diagram below, $\overline{AC} \parallel \overline{DF} \parallel \overline{BH}$ and $\overline{CB} \parallel \overline{FE}$.



a. Find four similar triangles. Explain how you know that they are all similar.

b. Using the similar triangles you found in part (a), complete the following extended proportion:

$$\frac{AB}{AC} = \frac{DE}{\blacksquare} = \frac{\blacksquare}{DG} = \frac{\blacksquare}{\blacksquare}$$

BIG idea Similarity

Lines with special relationships to the sides and angles of a triangle determine proportional segments. When you know the lengths of some of the segments, you can use a proportion to find an unknown length.

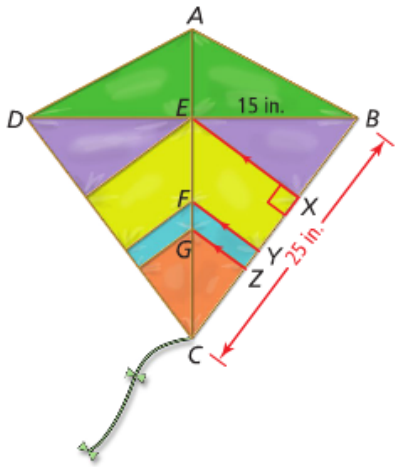
Task 2

You are making the kite shown at the right from five pairs of congruent panels. In parts (a)–(d) below, use the given information to find the side lengths of the kite's panels.

$ABCD$ is a kite.
$EB = 15$ in., $BC = 25$ in.
The extended ratio $XY : YZ : ZC$ is $3 : 1 : 4$.
$\overline{EX} \perp \overline{BC}$, $\overline{EX} \parallel \overline{YF} \parallel \overline{GZ}$

a. $\triangle BEX$ **b.** $XEFY$

c. $YFGZ$ **d.** $\triangle ZGC$



7 Chapter Review

Connecting BIG ideas and Answering the Essential Questions

1 Similarity
You can set up and solve proportions using corresponding sides of similar polygons.

Ratios and Proportions (Lesson 7-1)
The Cross Products Property states that if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

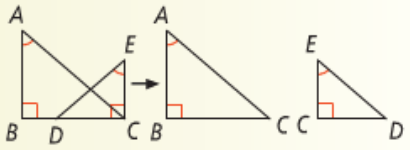
Similar Polygons (Lesson 7-2)
Corresponding angles of similar polygons are congruent, and corresponding sides of similar polygons are proportional.

2 Reasoning and Proof
Two triangles are similar if certain relationships exist between two or three pairs of corresponding parts.

Proving Triangles Similar (Lesson 7-3)
Angle-Angle Similarity (AA ~) Postulate
Side-Angle-Side Similarity (SAS ~) Theorem
Side-Side-Side Similarity (SSS ~) Theorem

3 Visualization
Sketch and label triangles separately in the same orientation to see how the vertices correspond.

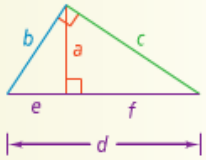
Seeing Similar Triangles (Lessons 7-3 and 7-4)



$\triangle ABC \sim \triangle ECD$

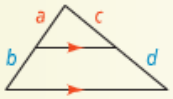
Proportions in Triangles (Lessons 7-4 and 7-5)

Geometric Means in Right Triangles



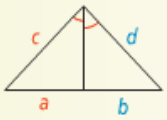
$\frac{e}{a} = \frac{a}{f}$ $\quad \frac{d}{b} = \frac{b}{e}$ $\quad \frac{d}{c} = \frac{c}{f}$

Side-Splitter Theorem



$\frac{a}{b} = \frac{c}{d}$

Triangle-Angle-Bisector Theorem



$\frac{a}{b} = \frac{c}{d}$

Chapter Vocabulary

- extended proportion (p. 440)
- extended ratio (p. 433)
- extremes (p. 434)
- geometric mean (p. 462)
- indirect measurement (p. 454)
- means (p. 434)
- proportion (p. 434)
- ratio (p. 432)
- scale drawing (p. 443)
- scale factor (p. 440)
- similar figures (p. 440)
- similar polygons (p. 440)

Choose the correct term to complete each sentence.

1. Two polygons are ? if their corresponding angles are congruent and corresponding sides are proportional.
2. A(n) ? is a statement that two ratios are equal.
3. The ratio of the lengths of corresponding sides of two similar polygons is the ?.
4. The Cross Products Property states that the product of the ? is equal to the product of the ?.

7-1 Ratios and Proportions

Quick Review

A **ratio** is a comparison of two quantities by division. A **proportion** is a statement that two ratios are equal. The **Cross Products Property** states that if $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $ad = bc$.

Example

What is the solution of $\frac{x}{x+3} = \frac{4}{6}$?

$6x = 4(x + 3)$	Cross Products Property
$6x = 4x + 12$	Distributive Property
$2x = 12$	Subtract $4x$ from each side.
$x = 6$	Divide each side by 2.

Exercises

5. A high school has 16 math teachers for 1856 math students. What is the ratio of math teachers to math students?

6. The measures of two complementary angles are in the ratio 2 : 3. What is the measure of the smaller angle?

Algebra **Solve each proportion.**

7. $\frac{x}{7} = \frac{18}{21}$

8. $\frac{6}{11} = \frac{15}{2x}$

9. $\frac{x}{3} = \frac{x+4}{5}$

10. $\frac{8}{x+9} = \frac{2}{x-3}$

7-2 and 7-3 Similar Polygons and Proving Triangles Similar

Quick Review

Similar polygons have congruent corresponding angles and proportional corresponding sides. You can prove triangles similar with limited information about congruent corresponding angles and proportional corresponding sides.

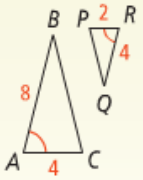
Postulate or Theorem	What You Need
Angle-Angle (AA ~)	two pairs of ≅ angles
Side-Angle-Side (SAS ~)	two pairs of proportional sides and the included angles ≅
Side-Side-Side (SSS ~)	three pairs of proportional sides

Example

Is $\triangle ABC$ similar to $\triangle RQP$? How do you know?

You know that $\angle A \cong \angle R$.

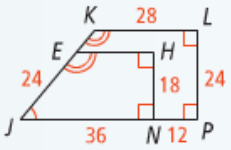
$\frac{AB}{RQ} = \frac{AC}{RP} = \frac{2}{1}$, so the triangles are similar by the SAS ~ Theorem.



Exercises

The polygons are similar. Write a similarity statement and give the scale factor.

11.



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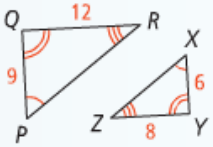
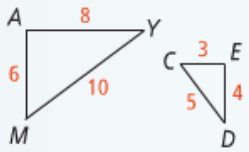


13. **City Planning** The length of a rectangular playground in a scale drawing is 12 in. If the scale is 1 in. = 10 ft, what is the actual length?

14. **Indirect Measurement** A 3-ft vertical post casts a 24-in. shadow at the same time a pine tree casts a 30-ft shadow. How tall is the pine tree?

Are the triangles similar? How do you know?

15.



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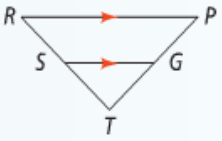
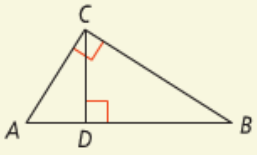


7-4 Similarity in Right Triangles

Quick Review

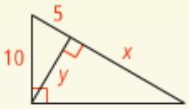
$\overline{CD}$ is the altitude to the hypotenuse of right $\triangle ABC$.



- $\triangle ABC \sim \triangle ACD$, $\triangle ABC \sim \triangle CBD$, and $\triangle ACD \sim \triangle CBD$
- $\frac{AD}{CD} = \frac{CD}{DB}$, $\frac{AB}{AC} = \frac{AC}{AD}$, and $\frac{AB}{CB} = \frac{CB}{DB}$

Example

What is the value of x?



$\frac{5 + x}{10} = \frac{10}{5}$	Write a proportion.
$5(5 + x) = 100$	Cross Products Property
$25 + 5x = 100$	Distributive Property
$5x = 75$	Subtract 25 from each side.
$x = 15$	Divide each side by 5.

Exercises

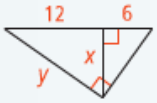
Find the geometric mean of each pair of numbers.

17. 9 and 16

18. 5 and 12

Algebra **Find the value of each variable. Write your answer in simplest radical form.**

19.



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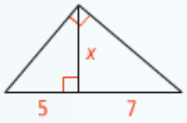

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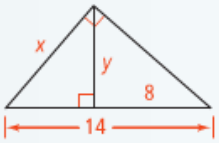

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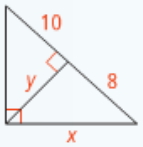


7-5 Proportions in Triangles

Quick Review

Side-Splitter Theorem and Corollary

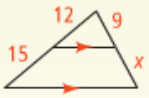
If a line parallel to one side of a triangle intersects the other two sides, then it divides those sides proportionally. If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.

Triangle-Angle-Bisector Theorem

If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that are proportional to the other two sides of the triangle.

Example

What is the value of x?

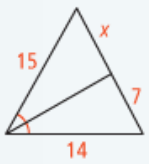


$\frac{12}{15} = \frac{9}{x}$	Write a proportion.
$12x = 135$	Cross Products Property
$x = 11.25$	Divide each side by 12.

Exercises

Algebra **Find the value of x.**

23.



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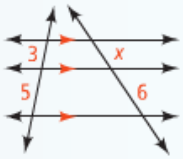

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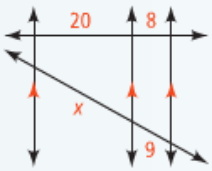

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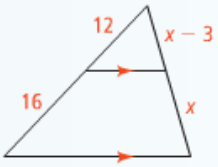

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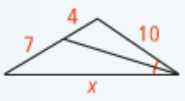

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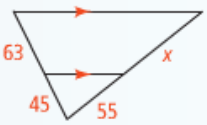

7 Chapter Test

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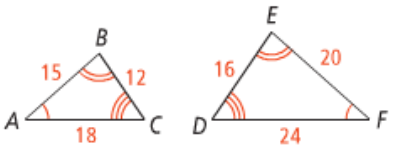
Do you know HOW?

Algebra Solve each proportion.

1. $\frac{x}{3} = \frac{8}{12}$

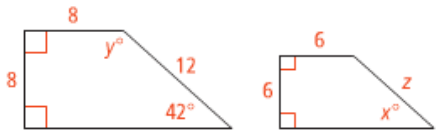
2. $\frac{4}{x+2} = \frac{16}{9}$

3. Are the polygons below similar? If they are, write a similarity statement and give the scale factor.



Algebra The figures in each pair are similar. Find the value of each variable.

4.



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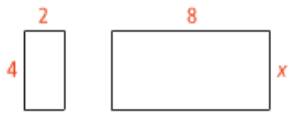

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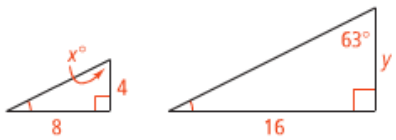
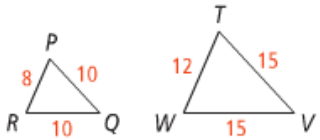


Determine whether the triangles are similar. If so, write a similarity statement and name the postulate or the theorem you used. If not, explain.

7.



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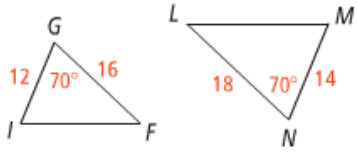


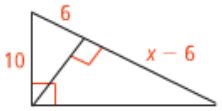
9. **Indirect Measurement** A meter stick perpendicular to the ground casts a 1.5-m shadow. At the same time, a telephone pole casts a shadow that is 9 m. How tall is the telephone pole?

10. **Photography** A photographic negative is 3 cm by 2 cm. A similar print from the negative is 9 cm long on its shorter side. What is the length of the longer side?

11. What is the geometric mean of 10 and 15?

Algebra Find the value of x.

12.



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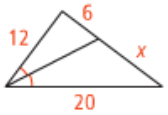

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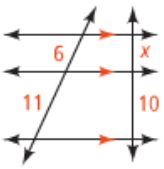

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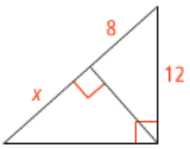
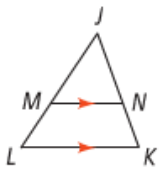


Do you UNDERSTAND?

16. **Reasoning** In the diagram, $\overline{MN} \parallel \overline{LK}$. Is $\frac{JM}{ML}$ equal to $\frac{MN}{LK}$? Explain.

17. **Reasoning** $\square ABCD \sim \square PQRS$. $\overline{DB}$ is a diagonal of $\square ABCD$ and $\overline{SQ}$ is a diagonal of $\square PQRS$. Is $\triangle BCD$ similar to $\triangle QRS$? Justify your reasoning.



Determine whether each statement is *always*, *sometimes*, or *never* true.

18. A parallelogram is similar to a trapezoid.

19. Two rectangles are similar.

20. If the vertex angles of two isosceles triangles are congruent, then the triangles are similar.

7 Cumulative Test Prep

TIPS FOR SUCCESS

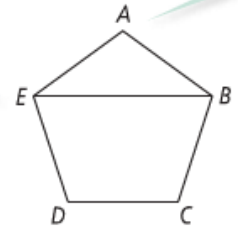
Some test questions ask you to find the measure of an interior or exterior angle of a polygon. Read the sample question at the right. Then follow the tips to answer it.

TIP 1

List what you know about $\triangle ABE$.

- $\overline{AE} \cong \overline{AB}$ because the pentagon is regular.
- $\angle ABE \cong \angle AEB$ because $\triangle ABE$ is isosceles.
- $m\angle A + m\angle ABE + m\angle AEB = 180$

In the figure below, $ABCDE$ is a regular pentagon. What is the measure, in degrees, of $\angle ABE$?



(A) 36 (C) 72

(B) 54 (D) 108

TIP 2

Find $m\angle A$ and then use it to find $m\angle ABE$.

Think It Through

By the Polygon Angle-Sum Theorem, the sum of the interior angle measures of $ABCDE$ is $(5 - 2)180 = 3(180) = 540$. So $m\angle A = \frac{540}{5} = 108$. Then $108 + m\angle ABE + m\angle AEB = 180$. Since $\angle ABE \cong \angle AEB$, $108 + 2 \cdot m\angle ABE = 180$. So $m\angle ABE = \frac{180 - 108}{2} = \frac{72}{2} = 36$. The correct answer is A.

Vocabulary Builder

As you solve test items, you must understand the meanings of mathematical terms. Match each term with its mathematical meaning.

A. corollary

B. geometric mean

C. midsegment

D. scale

I. the ratio of a length in a scale drawing to the actual length

II. a segment connecting the midpoints of two sides of a triangle

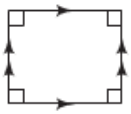
III. a statement that follows immediately from a theorem

IV. for positive numbers a and b, the positive number x such that $\frac{a}{x} = \frac{x}{b}$

Multiple Choice

Read each question. Then write the letter of the correct answer on your paper.

1. What is a name for the quadrilateral below?



I. square

II. rectangle

III. rhombus

IV. parallelogram

(A) I only (C) II and IV

(B) IV only (D) I, II, and IV

2. In which point do the bisectors of the angles of a triangle meet?

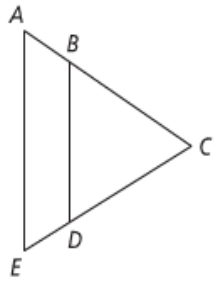
(F) centroid (H) incenter

(G) circumcenter (I) orthocenter

3. Which quadrilateral does NOT always have perpendicular diagonals?

(A) square
(B) rhombus
(C) kite
(D) isosceles trapezoid

4. Which of the following facts would be sufficient to prove $\triangle ACE \sim \triangle BCD$?

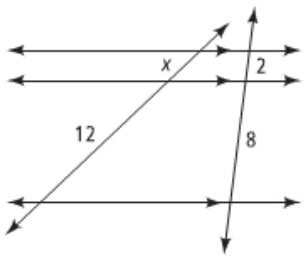


(F) $\triangle BCD$ is a right triangle.
(G) $\overline{AB} \cong \overline{ED}$
(H) $m\angle A = m\angle E$
(I) $\overline{AE} \parallel \overline{BD}$

5. What is the midpoint of the segment whose endpoints are $M(6, -11)$ and $N(-18, 7)$?

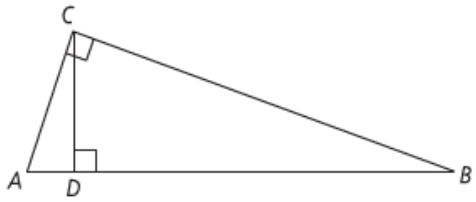
(A) $(-6, -2)$
(B) $(6, 2)$
(C) $(-12, 9)$
(D) $(12, -9)$

6. Use the figure below. By which theorem or postulate does $x = 3$?



(F) SAS Postulate
(G) If three parallel lines intersect two transversals, then the segments intercepted on the transversals are proportional.
(H) Opposite sides of a parallelogram are congruent.
(I) If two lines are parallel to the same line, then they are parallel to each other.

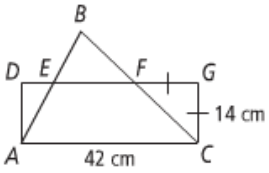
7. Which angle is congruent to $\angle DCB$?



(A) $\angle B$
(B) $\angle CDB$
(C) $\angle A$
(D) $\angle ACD$

8. In the figure below, $\overline{EF}$ is a midsegment of $\triangle ABC$ and $ADGC$ is a rectangle. What is the area of $\triangle EDA$?

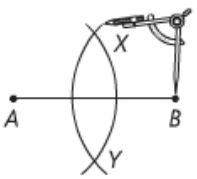
(F) 49 cm^2
(G) 98 cm^2
(H) 294 cm^2
(I) 588 cm^2



9. Andrew is looking at a map that uses the scale 1 in. = 5 mi. On the map, the distance from Westville to Allentown is 9 in. Which proportion CANNOT be used to find the actual distance?

(A) $\frac{1 \text{ in.}}{5 \text{ mi}} = \frac{9 \text{ in.}}{d}$
(B) $\frac{5 \text{ mi}}{d} = \frac{9 \text{ in.}}{1 \text{ in.}}$
(C) $\frac{d}{9 \text{ in.}} = \frac{5 \text{ mi}}{1 \text{ in.}}$
(D) $\frac{5 \text{ mi}}{1 \text{ in.}} = \frac{d}{9 \text{ in.}}$

10. What type of construction is shown below?



(F) angle bisector
(G) perpendicular bisector
(H) congruent angles
(I) congruent triangles

11. A student is sketching an 11-sided regular polygon. What is the sum of the measures of the polygon's first five angles to the nearest degree?

(A) 147
(B) 720
(C) 736
(D) 1620

12. Which of the following conjectures can be disproved with a counterexample?

Ⓕ The product of any two even integers is even.

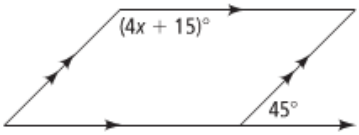
Ⓖ The sum of any two prime numbers is even.

Ⓗ Any integer divisible by nine is divisible by three.

Ⓘ The measure of an obtuse angle must be between 90 and 180.

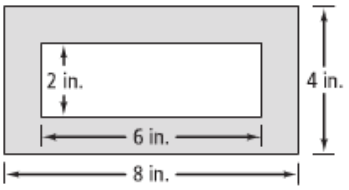
GRIDDED RESPONSE

13. What is the value of x in the figure below?

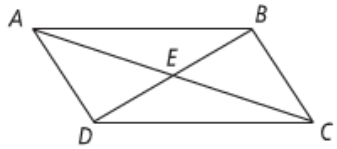


14. In hexagon $ABCDEF$, $\angle A$ and $\angle B$ are right angles. If $\angle C \cong \angle D \cong \angle E \cong \angle F$, what is the measure of $\angle F$ in degrees?

15. A scale drawing of a swimming pool and deck is shown below. Use the scale 1 in. = 2 m. What is the area of the deck in square meters?

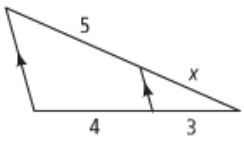


16. In parallelogram $ABCD$ below, DB is 15. What is DE?



17. The measure of the vertex angle of an isosceles triangle is 112. What is the measure of a base angle?

18. What is the value of x in the figure below?



Short Response

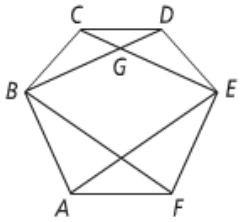
19. In rectangle $ABCD$, $AC = 5(x - 2)$ and $BD = 3(x + 2)$. What is the value of x? Justify your answer.

20. Draw line m with point A on it. Construct a line perpendicular to m at A. What steps did you take to perform the construction?

21. Petra visited the Empire State Building, which is approximately 1454 ft tall. She estimates that the scale of the model she bought is 1 in. = 12 ft. Is this scale reasonable? Explain.

Extended Response

22. In the diagram, $AB = FE$, $BC = ED$, and $AE = FB$.



a. Is there enough information to prove $\triangle BCG \cong \triangle EDG$? Explain.

b. What one additional piece of information would allow you to prove $\triangle BCD \cong \triangle EDC$? Explain.

c. What can you conclude from the diagram that would help you prove $\triangle BAF \cong \triangle EFA$?

d. In part (c), is $\triangle BAF \cong \triangle EFA$ by SAS or SSS? Explain.

23. At a campground, the 50-yd path from your campsite to the information center forms a right angle with the path from the information center to the lake. The information center is located 30 yd from the bathhouse. How far is your campsite from the lake? Show your work.